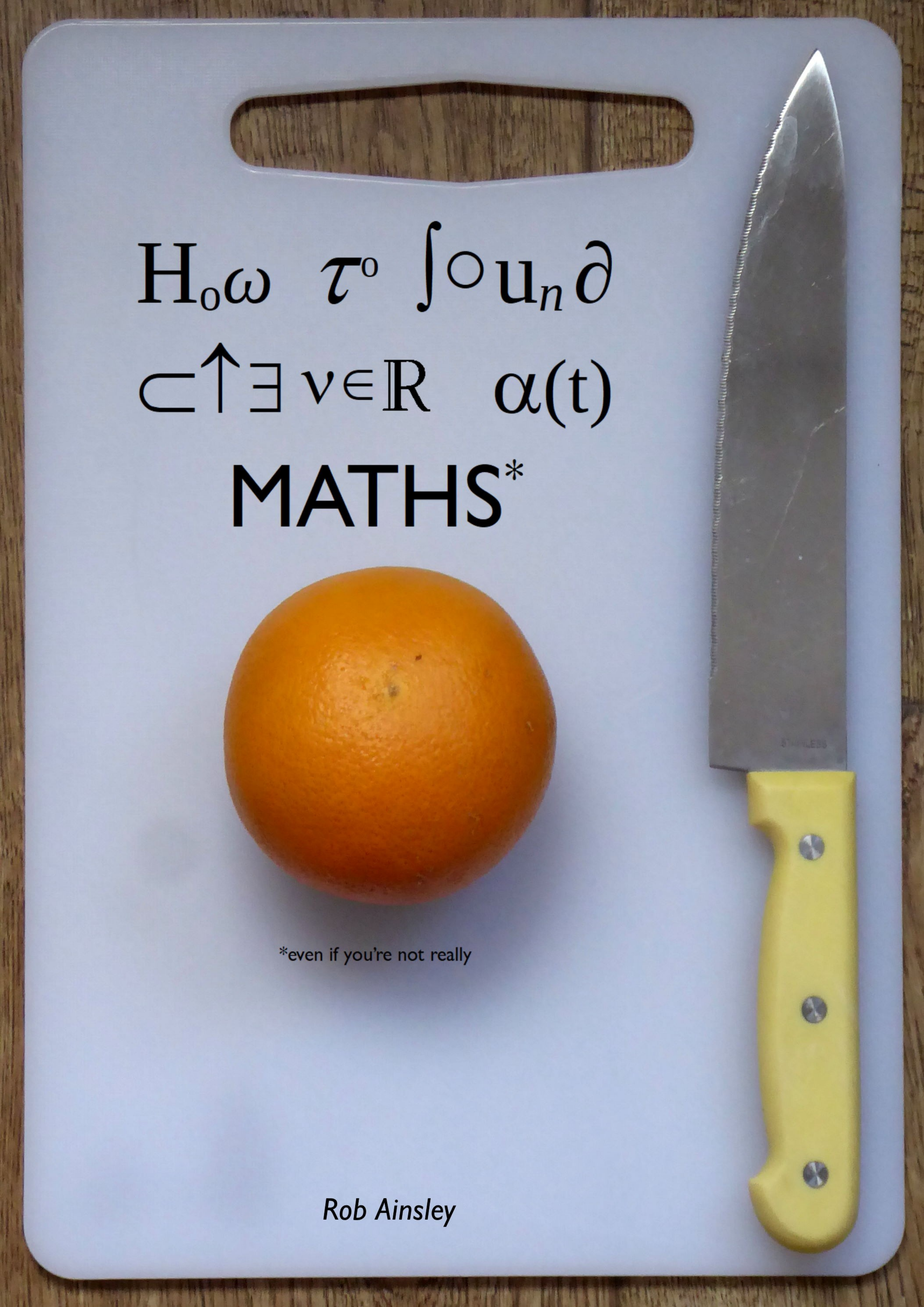

$\mathrm{H}_o\omega$ τ^{o} $\int\!\circ\,\mathrm{u}_n\partial$ $\subset\uparrow\exists\ \nu\in\mathbb{R}$ $\alpha(t)$ MATHS*

*even if you're not really

Rob Ainsley

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In memoriam Peter Neumann

Contents

CHAPTER 0: INTRODUCTION

Maths[1] – for decades considered obscure, dusty and irrelevant – is now hip and trendy. Who'd have predicted that?

Well, mathematicians, obviously, using probability theory. And algorithms. And data. All the stuff, indeed, that controls our lives now, deciding what Facebook posts to show you, whether or not to approve your mortgage, or which terrorist you resemble at check-in.

And mathematicians are now hip and trendy too. Geek is chic, and they don't come geekier than mathematicians. In the past we were unstylishly-dressed, hard to understand, bumbling-smiling people pitied and mildly ridiculed. Not today. Now we're unstylishly-dressed, hard to understand, bumbling-smiling people admired and mildly feared. And yes, guiltily fancied.

Because now maths, and mathematicians, shape the world. Not politicians, not big business, not the media. They need computers to do everything. Computers need programmers. Programmers need algorithms. And mathematicians control the algorithms. Nobody has a clue what's actually going on, except us. And even we're not sure most of the time.

A maths degree used to be a sentence of solitary confinement. Now you get to program music computers, devise financial models to see how much money you want to make for yourself, instruct terrified governments, and best of all, run across vats of custard with your friends.

Popular books on the subject abound. YouTube has some brilliant videos explaining everything. (And some boring ones.) Media mathematicians pop up on radio and TV, making the complicated sound simple and the simple sound clever, all with the laid-back smile of those who know our time has come, and who know the interviewer daren't interrupt because they're afraid of sounding stupid. Usually with good reason.

Maths is now mainstream. There are maths rappers, maths garden designers, maths musicians, maths stand-up comedians, maths jugglers, maths poets. Put 'maths' in front of anything and you sound a cut above. Maybe there are maths barbers, too.

And maths is the one thing we can rely on nowadays. Every other human intellectual endeavour is based on faith: something hopefully believed, something that usually works, but something ultimately unprovable.

Not maths. In a world increasingly full of hype, sell, spin, and other four-letter words, it's the only genuine, provable truth there is. It can even prove when it can't be proved. And we know that for a fact.

There's something divine about its abstract purity. God is, ultimately, a mathematician. It's just a shame She hasn't let on either way about the Riemann Hypothesis yet.

But to most people maths has a dark power. They're scared of it. They'll happily listen to music despite having no idea of the underlying theory[2]. But if, while explaining to them the best strategy for winning at Monopoly, you deploy a few simple keywords like 'x', or 'infinity', or 'stochastic simulation algorithm for Bayesian inference', they'll panic.

You can use this effect to your advantage. And you don't have to understand any equations or algebra. Only *appear* to. Have even the vaguest idea of how data encryption works, or why tests to identify potential terrorists are mathematically doomed to fail, or why buses all arrive at once, or why Trafalgar Square[3] is a good property to have[4], and you'll sound confident and clever.

You will outwit managers, sales staff, colleagues, friends, enemies, social media adversaries, TV and radio interviewers. If they think you understand some mathematical principle and they don't – no matter that you're in truth almost as clueless – you have a hold over them. *They* won't dare interrupt you either.

In short, this book will tell you how to sound clever at maths, even if you're not really. Or, as a mathematician might put it:

> *Let a_p be the actual maths ability of a person p, and $f(a_p)$ their apparent maths ability after reading this book. Then, for any non-mathematician q, $f(a_p) > a_p$ and $f(a_p) > a_q$ for all $a_p \geq 0$.*

See? Well, it doesn't matter if not. All that's saying is, once you've read this book, your apparent maths ability will be more than your actual ability. But even better, anyone who isn't already a mathematician will think that *you* know more than *they* do. However much you actually do. (Indeed – as you possibly spotted – the amount you actually know might be zero.)

It just says all that in a flashier way. A more mathematical way. One that's more rigorous and more useful. Particularly for intimidating non-mathematicians. Or, as they're sometimes called, 'normal people'.

And the neat thing is, you don't need to worry if you encounter a real mathematician. They'll celebrate your new-found ability to bluff as well as they do, and treat you as an equal. Or, at least, as someone homeomorphic[5] to them. (That's a compliment.)

In the past, we mathematicians struggled to live in everyone else's world. Today, everyone else struggles to live in ours. Don't worry. We'll tell you how to survive.

Welcome to maths.

CHAPTER 1: WHAT IS MATHS?

Proof it's now cool. Proof it's the best. Proof you can't prove anything.

Many questions in mathematics have not yet been satisfactorily answered. What's the largest sofa you could possibly shift round an L-shaped corner[6]? In a group of 45 people, must there always be five mutual friends or five mutual strangers[7]? Can a computer solve any problem, in principle[8]? Do all the nontrivial zeroes of the Riemann zeta function have real part ½[9]? How come shared restaurant bills never add up?[10]

But, trickiest of all, *what is mathematics?*

There are various answers. For children, 'maths' might be, simply, 'adding up and multiplying'. For schoolkids it might be quadratic equations, of no apparent relevance to real life[11]. For university students it might be a way to get a better job on graduating. For working-age people it might be the black arts behind computer algorithms that bump up the price of the flight, or display ads on their Facebook page for a lawnmower when they've just bought one. For the retired, watching *Countdown*[12] on British TV, it might be 'adding up and multiplying'.

It's certainly not 'hard sums'. The further you go down the maths rabbit-hole, the fewer numbers – and the more letters of the alphabet, strange typefaces, and bespoke squiggles – you see. It comes down to concepts, not numbers. (Unless you're doing number theory, of course.) Now that nobody has to recall phone numbers because they're stored on your mobile, professional mathematicians may go months without seeing any familiar numbers apart from 0, 1 and infinity.

Maths is all this and more. Infinitely more, in fact. To the smart, well-informed individual, which now includes you, mathematics is no less than the language of creation.

Ask a silly question

So what *is* it? In essence, maths is about asking silly questions. And finding clever answers.

It's about abstraction, seeing the underlying truth of things. About not getting bogged down in the real world's irrelevant details. Preferably avoiding the real world entirely.

It's about the background: standing back to see the big picture, in a way that then enables you to understand the patterns. Then you can make useful real-life predictions about how the patterns will work out in new situations. Translating something absurd into something deep and meaningful. Such as doing it an infinite number of times, in four dimensions, backwards.

Once you've got over the basics, like counting your sheep and doing your tax returns, the interesting stuff begins. What do 'minus two sheep' look like? How long is the side of a field of negative area? Does zero divided by zero mean anything? Can you multiply two by itself one and a half times? What would a two-and-a-quarter-dimensional shape look like? If there's a set of all things that don't contain themselves, does that set contain itself?[13] Would aliens have maths, perhaps even 'better' maths than ours, which could understand British railway ticket pricing?

Branches of maths

Algebra is 'the language of maths' – *x*s and *y*s, Greek letters, numbers, equals signs, arrows, squiggles, curly brackets, backwards 6s etc. It includes many branches such as G*roup Theory*[14], *Graph Theory*[15], *Topology*[16] etc, which are good places to avoid numbers.

Arithmetic is stuff you could do on a calculator.

Geometry measures things that stand still.

Analysis measures things that move.

Probability measures how often something happens in the long run. **Statistics** measures how surprised you are that it just did.

Pure and applied maths

For a pure mathematician, pure maths is – well, the clue's in the title – the real deal, what proper maths is all about. It's about how things *actually work*, in theory, before the real world and real people spoil everything. Applied maths, with its over-long and complex equations spilling over several lines, is messy, makeshift, and not beautiful or elegant. Maybe your plane flew OK this time, but if you don't understand why, what's to say it will next time? Pure mathematicians tell you *maths is a language*.

For an applied mathematician, applied maths is – well, the clue's in the title – the real deal, what proper maths is all about. It's about how things *actually work*, in practice, not in some idealistic dream-world. Pure maths, with its over-long and complex equations spilling over several lines, is all very elegant and beautiful, but it's better that a plane looks ugly and stays airborne. Applied mathematicians tell you *maths is a toolbox*.

'Maths is a language'

True to some extent: not a language like Xhosa or Polish or Chinese say, and not on Google Translate, but a language for encoding, representing, understanding and predicting the way all of creation, and anything yet to be created, works.

Or at least, the way creation *should* work, if only reality didn't get in the way. It's an incredibly rich, powerful and universal language, independent of race, creed, colour or side of the Brexit argument, even if Xhosa is easier to pronounce, Polish is simpler to learn, and Chinese has a smaller set of symbols.

Over 99.9% of the time this language produces 'beautiful' (ie useless) results. But sometimes, the remaining 0.1% will turn out to have astoundingly useful (ie ugly) applications in the real world, in a totally unforeseen way. Unfortunately, at the time, we never know *which* 0.1%.

'Maths is a toolbox'

That useful 0.1% is the maths toolbox. For examples of recondite maths theorems from pre-computer days proving unexpectedly useful later on, try **Euler's Theorem** on prime numbers ('the basis of all internet security – without it, the world economy would fall apart'). Or **Shannon's Theorem** on encoding and correcting transmitted information ('*if it wasn't for Shannon, every time you copied a computer file, listened to a CD or downloaded a file over the internet, you'd get nonsense – well, worse nonsense, anyway*').

If you have to make an example up, try something like '*nobody's heard of the Lewandowski–Kane Theorem, yet it underpins the way GPS works*', or '*even mathematicians are unfamiliar with the Lukaku–Ibrahomović*[17] Equations, yet they could enable quantum computers'.

Wikipedia and the internet

You don't need to know what the aforementioned theorems, or any theorems, actually are. If anyone asks, give the smile of the adult asked by a child where money comes from or why there are wasps. It's complicated. It's hard to explain. But '*there's a good article on the topic on Wikipedia*', you say.

Because that's always true. Wikipedia is your friend. The maths pages are generally good, reliable, and explained about as well as can be, for the simple reason that only mathematicians have the ability and knowledge to edit them – unlike the buffoons who vandalise pages on say politics or history or your favourite band.

Look up YouTube videos. Channels such as Matt Parker, 3Blue1Brown, Numberphile and others do excellent jobs producing computer visualisations or practical demonstrations that vividly explain how even sophisticated maths concepts work. Pointing people in these directions is a useful tactic for you. They'll get distracted by YouTube's suggestions of other videos to watch – music, films, TV clips and so on. So they'll never get round to understanding enough to realise you were winging it. Because *you* never got to the end of the maths video either, before also being waylaid.

Who's fooling who?

If you're worried about giving the impression you understand more maths than you actually do, don't. That's what the professionals all do. Beyond undergraduate level, *all maths is bluffing*. There's simply so much maths out there, much of it hugely dense and complex, that the vast majority of ideas, theorems or conjectures are understood by only a handful of people. Outside their own niche, mathematicians have to take virtually everything on trust.

Imagine two biologists meeting at a conference, one a global guru in plankton, the other a world expert in elephant dung. They'd have little in common in terms of subject matter, and certainly couldn't challenge any opinion the other had. They'd have to take everything at face value.

Well, it's like that when even a pure mathematician and applied mathematician meet. Except that each thinks the other is the specialist in elephant dung.

And some universally accepted things aren't even understood by anybody: the theorem that says four colours is enough to colour any sensible map is so long, it has only been proved by a computer[18]. And *that* could be making things up.

Eternal verities

Unlike all other fields of human intellect, maths is forever, always true, never 'out of fashion'. Getting a tattoo of your partner's name or favourite band is a terrible idea: things might change ten years from now. But a tattoo of say '$e^{i\pi}+1=0$'[19], or the Julia set[20], is a good idea. It will remain true for all eternity, perhaps longer. The only problem is finding a tattooist with a mathematical character set, and infinitely small needles.

(However, don't be tempted to tattoo yourself entirely with patches of colour to demonstrate the four-colour map theorem. Because the human body is not topologically equivalent to a sphere. It's more like a torus[21], and that would require seven colours.)

Does maths really exist?

Or did humans just make it all up? Would aliens also have Pythagoras's Theorem, or the Goldbach Conjecture, or know the Blancmange Curve is continuous but not differentiable, even if they didn't have blancmange[22]?

Until we find their Twitter handle, we can't contact them to find out, but most of us suspect that maths – at least most of it, in principle – is actually *there*, waiting to be uncovered.

Or at least, we hope. Because in 1931, the spoilsport Austrian logician Kurt Gödel ingeniously (and relatively simply[23]) proved that no mathematical system can be complete. That is to say: however you set up the rules, there'll be things that are true – but which, frustratingly, you will *never be able to prove*. (Anyone who wonders where the chocolate disappeared and why their dog is looking shifty will know the feeling.) You can fix it by adding some more rules to your setup – but that only generates new unprovable things elsewhere.

So maybe maths is just a collection of arbitrary games?

Except there's just so much cool stuff, so many joyously weird, mind-blowing, trippy, crazy things we find in theory and *which then turn up in real life*, that we think maths *must* be true. Mere humans couldn't have invented anything as clever. Particularly when you see what humans say in comments online.

New dimensions

So don't worry if you don't understand much. Nobody does. Just enjoy the bizarre, enthralling, hallucinatory world of mathematics. A world where you can add up all the whole numbers and get $-{}^{1}/_{12}$. A world where some infinities are bigger than others. A world where you can chop a ball into bits and reassemble them into two of the same size. A world where numbers can be imaginary, perfect, deficient, amicable or sexy.

A world where, even if the bill still doesn't add up, we might be able to come up with a reason why it would – in a four-dimensional, complex, infinitely big restaurant.

CHAPTER 2: MATHEMATICIANS

How to spot them, befriend them, and if necessary, avoid them

'A mathematician is a machine for turning coffee into theorems', the old saying goes. But what are we like? How do we think? How can normal people learn to interact with us?

Start with the following story. An engineer, a physicist and a mathematician are on a train in Yorkshire. They spot a black sheep in a Dales field[24]. 'Wow! Sheep in Yorkshire are black!' exclaims the engineer. 'No', says the physicist, 'there's at least one sheep in Yorkshire which is black'. 'No', says the mathematician, 'there's at least one sheep in Yorkshire which is at least one-half black'.

The joke – which you can embellish, transplant, or rewrite with a different species – sums up mathematicians' image as pedants, out of tune with how normals think.

Actually, we're not pedantic, we'd reply: we're just *precise*. There's a difference, you know.

We tend to take rules and instructions literally. Take the shampoo bottle that says 'Directions for use: 1. Wet hair. 2. Apply shampoo. 3. Rinse and repeat.' The obedient mathematician will be doing that till the shampoo runs out.

Or take the announcement on trains to 'please keep all your belongings with you at all times'. It's hard, carting the contents of your wardrobe, book collection and furniture round with you.

It's true we have an image problem sometimes. We may talk a bit too fast to be followed easily. Our mind always seems to be on something else. We sometimes don't notice basic social interactions, or even the fact we just spilled coffee down someone's shirt, before eventually realising it's our own.

There's some truth in that. But it's not that we've, say, forgotten to buy our round in the pub. We're just trying to work out the fairest game-theory strategy for deciding who buys what each time.

Mathematicians admire people who don't admire themselves. Follow the logic and you can see that our self-esteem issues can therefore be complex. But we don't mind other people, generally. It's just that we like ideas a bit more. Because maths is reliable, and people aren't. Maths is consistent, or at least inconsistent in a consistent way. But people say they love you and then go off with an algebraic topologist instead.

Diversity, in terms of gender and ethnic opportunity, is pretty good among mathematicians. At least, we can genuinely claim not to have noticed that you're for example female, or Asian, or using a wheelchair. We certainly noticed that unusual differential equation you wrote down, though.

We're generally good people. We're relatively unlikely to murder anyone, for instance. Unless perhaps they're a Frequentist who's just slagged off our Bayesian-approach PhD thesis. Or a love-cheat algebraic topologist. But we'd be very interested in working out how to commit the perfect murder *in theory*. Not just murdering one person, but n people. And what happens when n is negative. Or real, or complex, or infinite.

How to befriend a mathematician

This one's easy. Just find a bit of maths you don't quite get, and ask us to explain it. We love doing that, especially if you can look genuinely interested.

Don't worry about coming across as dim. Any decent mathematician will be sympathetic. We know exactly what it's like simply not to understand something, or to think we've understood but then get it wrong, to actually understand something and still get it wrong, or to actually understand something and get it right but be unable to explain it to anyone else.

Like Columbus[25], we set out to investigate an area of maths not really sure where we're going. When we get there we've little idea where we are. And when we publish our findings, we don't know where we've been. And, ultimately, we find someone else got there first anyway.

How to get rid of a mathematician

Sometimes necessary. If you're a politician, for instance, and a maths researcher has inconveniently proved that your policy idea won't work.

But it's easy to make us go away. Don't say anything silly like 'Will you just go away?'. You'll get a genuinely innocent reply of 'Which direction? Two or three dimensions? Linear or curved path? Is it continuous? Can the path ever cross itself?' etc.

Instead, say something like '*Wow. Interesting. But I'm sure I saw a YouTube video that came up with a recent counterexample/ proof/ extension to 12 dimensions. Or maybe a tweet. Or a Facebook post? Instagram, perhaps...*'

We'll be off to follow your shoal of red herrings, and – you confidently hope – get waylaid by some other piece of maths we stumble across meantime.

How to seduce a mathematician

Our idea of closeness may not match that of non-mathematicians. During pandemics we all look forward to the two-metre social distancing rules being relaxed so that we can revert to our normal five metres.

So the usual eye contact, raised eyebrows and meaningful smiles may not work if you're trying to get intimate with us. We'll probably misread your signals, and think we've made a mistake, that you've come up with a clever counterexample to the statement we just made, or worry that we assumed a fraction like $^{28}/_{91}$ was reduced to its simplest form[26], or something.

And asking 'Can I ask you for a date?' may not work either. We'll look bewildered and say, 'What sort of date? Julian or Gregorian calendar? Or Islamic? Is there an upper or lower bound?' etc.

So you'll have to be more subtle. Appeal to us in a language we understand better.

For example, look up the Fussy Suitor Problem (aka Secretary Problem) on Wikipedia. '*You know the $^{1}/_{e}$ stopping rule for the Fussy Suitor Problem[27]? Well I think I might have just reached the place to stop, but I need your help to prove it...*'

Or say something like, '*I'm struggling with something. The Coase Theorem states that in the presence of externalities, if there are low transaction costs and free trade, then bargaining will lead to a Pareto-efficient outcome regardless of initial ownership or, in our case, relationship statuses. Are we in a kind of Prisoner's Dilemma here, or should I ask you out anyway?*'.

How to make a mathematician laugh

Our sense of humour is based on the deliberate mistake, non-sequiturs, the absurd, and the misuse of logic, because these things remind us of our PhD thesis. You'll quickly get on our side by saying such things with a smile, showing that you know exactly what you're doing.

Similarly, don't worry if we say similar things to you. We're not trying to catch you out; we're complimenting you, assuming that you're in on the joke, too. Examples:

> 'My favourite prime number? Oh, 91.'[28]
> 'This is a conjecture about perfect prime numbers.'[29]
> 'This formula shows the ideal number of children for a happy family: π/2.'[30]
> 'The survey asks people which of two types they are: those can be put into one of two types, or those that can't.'[31]

Don't worry if it takes a while to work some of these out. Comedy is about timing[32]; mathematicians just have a different sense of it. Maths is eternal, after all.

How to tell a joke to a mathematician

Precision is the key. Define your terms. Explain everything.

'*A horse, h, walks into a three-dimensional finite bar in which every Cauchy sequence converges. The barman says 'Why the face of length l(h) at least two standard deviations above the mean? And the horse says, "Equine face lengths are not normally distributed. A pint of IPA, please."*'

And so on. Otherwise, it just won't be funny.

Eccentric mathematicians

Are mathematicians really that different from 'normal' people? You can argue either way, depending on who you're trying to impress.

So, you can try the '*mathematicians are only as bonkers as anyone else' approach (with confident use of phrases like 'statistically...*' and '*contrary to popular misconception...*' and '*regression analysis shows that...*'). Most mathematicians are boringly 'normal', you say, but you don't notice those ones. As for the ones you *do* notice, the so-called 'eccentrics', it's not the maths itself, it's the environment they use the maths *in* which explains their behaviour.

In other words, any eccentricities of maths teachers are only those of other teachers. Maths professors are only as odd as any other professor. A male/ female mathematician is a bit weird because he's/ she's male/ female, etc. All those stories about 'eccentric mathematicians' are tropes, clichés, wild exaggerations, by people rubbish at maths desperate to reassure themselves they're better off being innumerate and 'normal', you say.

That said, some of us do non-standard things. A Google search for 'mathematician juggler unicyclist' brings up lots of results. But it's no use complaining we're eccentric just because we regularly wear only one sock[33], for instance. That's regular behaviour to some of us.

So, you'll have to define eccentricity in a language we can relate to. As the locus of points whose distances to a point (the focus) and a line (the directrix) are in a constant ratio, perhaps. Then you can quantify us: '*On a scale of 0 for a circle to 1 for a parabola, I'd say you're a hyperbola. With no upper bound.*' We'll probably still take that as a compliment.

Anyway, if you need to cite a few examples of mathematicians who behaved in bizarre ways, if only to stress how *few examples there actually are*, you have plenty to choose from. Here's a few.

Archimedes (?fl. 200 BCE) Early investigator of calculus-like methods who, on getting a brainwave about dunking things in water, supposedly jumped out his bath and ran through the streets of Syracuse naked shouting 'I've got it!'. Total myth, of course, but easier to remember than Archimedes' Principle itself.

Isaac Newton (1642–1727) Co-inventor of calculus who once stuck a needle in his eye to see what would happen, dabbled in alchemy and the occult, and served as an MP – though the only time he spoke in Parliament was to ask someone to close a window.

David Hilbert (1862–1943) German guru and maths mover–shaker who happily borrowed ladies' furs or feather boas when he was cold – a man whose creative dress sense, as well as mathematical vision, was ahead of his time.

Alan Turing (1912–1954) Computer theory trailblazer who would cycle round Bletchley Park with pyjamas under his clothes wearing a gas mask.

Paul Erdős (1913–1996) Published over 1500 papers across huge areas of maths and lived a homeless, itinerant-academic life out of a suitcase. His idea of a greeting was not 'Good morning', but 'Let *n* be an integer, and f(*n*) a function such that...'

Claude Shannon (1916–2001) Pioneer of information theory, juggler and unicyclist, who invented a flame-throwing trumpet.

Simon Norton (1952–2019) Group theorist and bus timetable expert whose personality quirks, including a non-standard approach to lavatory use, were the subject of the best-selling book *The Genius in my Basement*.

With the sheer rarity of eccentric mathematicians thus established, you can move on to more interesting stuff, like recommending YouTube popular maths channels. Singing Banana, for instance, presented by the engaging James Grime, academic, communicator, juggler and unicyclist.

CHAPTER 3: NUMBERS

Think of an unthinkable number – mathematicians have already imagined it

Maths is about far more than numbers. But it's useful to know some, such as 1, or 0, or 1,729. If you don't already know many numbers, you can make some up.

What is a number?

Numbers started as early humans' way of ensuring they hadn't lost any livestock, such as sheep or children, by ticking them off against notches on a bone. To reassure anyone worried about their hopelessness with numbers, talk about 'stone-age tribes' in places like the Amazon Basin or Papua New Guinea whose languages have no words for numbers beyond say 3 or 4[34]. By this, you imply that arithmetic is all some ghastly construct of modern man, somehow implicated in terrible consequences of progress such as deforestation, climate change, plastic pollution, YouTube comments etc.

The question of what a number *actually is* confounded mathematicians for millennia. Not until the early 1900s, when set theory was used to define numbers, were we certain that 1+1=2. It took Whitehead and Russell until page 379 of their *Principia Mathematica* to prove it. They got this down to page 360 in a later edition: mathematicians constantly strive to make their proofs more compact.

The set theory definition of numbers has a stunningly simple basis. 'Zero' is the empty set – the set of things with no members, such as four-sided triangles, or house sales that complete when the estate agent promised they would. 'One' is the set that contains the empty set – clearly there's just the one empty set, so we've got a unique definition of 'one'. 'Two' is the set that contains 'one' and the empty set – two of those, obviously – and so on for 'three', 'four', 'one thousand seven hundred and twenty-nine', etc.

If people are in danger of understanding this, draw a few diagrams of paper bags containing many nested levels of bags-within-bags. Ideally, *on* a paper bag. Write things like '2={0,1}={∅,{∅}}'[35], and say blithely '*you can easily see how it continues, and hey presto!, we've defined all natural numbers, and by extension, all fractions and decimals...*', and that should keep them quiet.

So, by sleight of hand, we've defined:

Zero You know this one. Can be trickier that it might seem ('*What's zero divided by zero?*[36] *Or zero to the power zero?*'[37] etc) but a number depressingly familiar to most of us.
Natural numbers What normal people call 'numbers': 1, 2, 3 etc. The set of them is referred to as $\mathbb{N}$ for 'natural', so they must be good for you. If you include zero and all the negatives then the set is called $\mathbb{Z}$, from the German 'Zahlen', 'numbers'. That curious semi-outline text style for the

numbers is called either ‘double-struck’ or ‘blackboard font’; if someone knows either of those terms, insist the other is the ‘more correct’ one.
Rationals Or ‘fractions’: anything with a / in it, such as ¾, or $^{355}/_{113}$ (or indeed $^{1729}/_{1}$). Called ℚ, from ‘quotient’, though you could try to make it’s because all fractions can actually be written out in order or ‘queued’, which would be quite a clever observation. (See types of infinities.)
Real numbers Aka ‘decimals’: any number you can write down with a point, not that mathematics always seems to have one, such as 0.001, or 6.25, or 98.7654321. Unlike the other numbers, though, these ones might go on for ever, hence can’t be completely written down. (And can’t be written out in order, either, unlike rationals). In that case, we stick on an ellipsis at the end, just like we do with other infinitely long lists. (For example, ‘the only problem with this politician is that we can’t trust what he says today, tomorrow, the day after, the day after that...’) Sometimes this might be simply the way we write it (for example, $^{1}/_{3}$ is exact, but 0.333... isn’t, though of course it repeats *ad infinitum*)[38] and others the number might go on forever (the square root of 2 is 1.414213... but never repeats). The reals are referred to as the set ℝ, for ‘real’. Really.

Each set contains the one below: all natural numbers are rational and all rationals are real, but not vice-versa. Perhaps one day a grass-roots movement will start a Campaign for Real Numbers, following similar initiatives for Ale, Tennis, Madrid and so on.

Fashionable squares, prime property

There’s a bewildering list of ways mathematicians describe numbers, which make them sound like they have personalities. (Speculate on *why* we do this if you want to move the conversation away from actual numbers; everyone will have an opinion.)

For instance, a ‘perfect’ number is one that equals the sum of the numbers that divide into it (eg 28=14+7+4+2+1). A ‘deficient’ one equals more than the sum of its divisors (eg 16 is more than 8+4+2+1, which only adds up to 15). And so on: numbers can be ‘amicable’[39], ‘friendly’, sociable’, ‘happy’, ‘narcissistic’ etc; make up some more if you like.

However, there’s only two sorts of number you really need to know: squares and primes.

Squares are, obviously, the result of multiplying something by itself, such as 4 (2x2) or 25 (5x5) or 2,989,441 (1729x1729).

Primes are numbers which no number divides nicely into, except 1. They lie behind patterns all over the place, the maths equivalent of elements in chemistry’s periodic table, but there’s an infinite number of them and they maddeningly lack any obvious structure: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37... but later on, ...997, 1009, 1013, 1019, 1021, 1031, 1033, 1039...

If a large box contains a prime number of chocolates, you can’t share them fairly between any possible number of people: there’ll always be some left over. Unless of course you have them all yourself. So what more proof do you need.

You're negative, I'm just complex

Somewhere in or after the Little Ice Age, mathematicians came up with the idea of zero sheep, even negative sheep. Even though nobody could quite visualise 'minus three ewes', negative numbers proved useful for accounting purposes, and for temperatures, explaining why it felt so damn cold.

Lots of people still don't get zero or negatives. Sound superior about this: it's evidence of how dim most people are.

For instance, railway stations with a Platform 0, such as Kings Cross or Leeds, often have to explain to puzzled commuters that 'Platform 0 is next to Platform 1'.

Or try this: there was never a 'year zero' in the Christian calendar, which jumps from -1 (or '1BCE') to 1 (or '1CE') without a zero in between, which profoundly annoys mathematicians, even those of zero religious beliefs.

Or this: the ground floor in US buildings tends to be numbered 1, but the basement -1, again leaving no 'floor zero'. This also profoundly annoys mathematicians, even those who never get to stay in nice hotels.

Or this: a winter-themed UK scratchcard that won you a prize if you revealed a lower temperature than the marked one bewildered thousands of people who refused to believe that -4C is warmer than -5C.

However, things get worse. The squares of -2 and +2 are both +4, so what's the square root of -4?

To do this, mathematicians visualised numbers as if on a big horizontal thermometer, or perhaps railway station platforms, with zero in the middle, negatives going off to the left, and positives to the right.

Then, cunningly, they added a whole extra 'number line' going vertically up and down – popularly called 'imaginary numbers'. These have the property that squaring them moves them round from vertical to horizontal, which is hard to picture in terms of railway platforms. The square root of -1 is *i*, so the square root of -4 is 2*i* ('i' for 'imaginary'). Problem solved!

It looks a fix, but it works in all kinds of neat ways. 'Complex numbers' – ones with a 'vertical' and 'horizontal' component, which look like '2+3*i*' and so on – are fantastically useful tools in maths. Despite the fact they seem to have no link with reality, they handily describe things like electric currents, for example when your trains are delayed from Kings Cross by cable problems.

Should anybody appear to have followed you so far, go more obscure. Move on to 'quaternions' – triply-complex numbers that sort-of add yet more dimensions to the 'number line'. Here you have not just i but also j and k, and $i^2=j^2=k^2=ijk=-1$. They were dreamed up by the Irish mathematician William Hamilton in 1843. Waffle about how they are 'useful in quantum physics' and 'handy for coding computer graphics', but more memorably, how Hamilton was so pleased with his idea, which came to him as he walked along Dublin's Royal Canal, that he graffitied it on to a bridge.

If people know about *that*, you'll have to blind them with mentions of 'octonions' (eight dimensions to quaternions' four) and 'sedenions' (16 dimensions) and vague talk about 'possible applications in quantum computing/ fundamental theories of atomic particles/ time travel using black holes' etc.

Good numbers to know

Be reasonably familiar with some numbers: 0, 1 and 2 are a good start, and any number you can think of beyond that will help. It helps to know that you can't divide by zero; and that some numbers which look prime aren't, like 91; and that a million is 1,000,000, a billion is 1,000,000,000, and a trillion is 1,000,000,000,000. You don't really need familiarity with anything bigger unless you're an astronomer or have a bank account in {country with hyperinflation that won't offend your audience}.

A one followed by a hundred zeros (denoted as 10^{100}) is a 'googol', named by a mathematician's nine-year-old-nephew in 1920. The similarity to the title of the search engine Google is not accidental: founders Sergey Brin and Larry Page used the name to suggest the vast quantities of data (and, subsequently, money) they were compiling, but got the spelling wrong.

Proofs occasionally require ludicrously large numbers which might be worth noting. Graham's Number, for example – denoted g_{64} – was once touted as the 'biggest number in maths', but is so vast it can't be described in any meaningful way. (Helpfully, though, we know it ends with a 7.) At least it's easy to bluster about: '*Indescribable in physical reality... more than the number of particles in the universe, even if each particle contained a universe, and so on for a number of times equal to particles in the universe... can only be described with Knuth's up-arrow notation... so big that if you succeeded in imagining it, your mind would literally be blown, because your brain would collapse entropically into a black hole, no not figuratively, that's literally what physics predicts...*' etc.

There are, however, even higher numbers nowadays. The most celebrated, and easiest to remember, is TREE(3). It's the maximum number of possible unique networks generated by joining dots of three colours that follow certain simple rules. TREE(1) is 1, TREE(2) is 3, but TREE(3) is ludicrously big. ('*Makes Graham's Number look like a PIN code... related to Kruskal's Theorem, of course... even proving TREE(3) is finite with pen-and-paper maths would take longer than all possible states of the universe, so existence would reset itself before you could finish...*'). You can could make up more, such as TREE(4), or TREE(TREE(3)), or TREE(TREE(TREE(3))), or FOREST(n) perhaps, defined as 'TREE(n) nestings of TREE(n)'. However, the real mathematician stays cool here, with a smile and shrug that says you won't waste too much time on such games. ('Shows the power of maths to transcend existence... glimpse into mind of God... infinity... life... death...', etc)

The only specific biggish number you really need remember though is 1,729, because it features in a story well-known to mathematicians. (It has its own Wikipedia page, which helpfully introduces it as 'the natural number following 1,728 and preceding 1,730'.)

That's because it is the 'Ramanujan Number', after the great maverick number-theorist Srinivasa Ramanujan. Visiting him in hospital, the English maths don GH Hardy remarked that he had arrived in taxi no. 1729, calling it a dull number. "On the contrary", smiled the ill Ramanujan, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways".

You can use this as a test of the person you're telling the story to. If they look puzzled, they're quite normal. If they laugh, they've read some popular maths books. If they frown, grab a pen and paper, and after some thought, scribble down '$1729 = 1^3 + 12^3 = 9^3 + 10^3$', they're *genuinely* a mathematician.

As for non-whole numbers, the only ones to know are:

π 'pi', 3.141592653589793[40]..., going on forever and never repeating, though that doesn't stop some people programming computers to try and print it all out.[41] Well-known as the ratio of a circle's circumference to its diameter, though you can say it '*crops up all over mathematics almost mystically*', citing things such as the following, and ending with '*how does that work, eh!*': the probability that two randomly selected numbers have no common factor; or the sinuosity of a winding river; or Buffon's needle problem, an '*experiment that involves dropping needles and seeing if you end up treading on them, which can be used to calculate π...'* To lead the conversation away, talk about 'Pi Day' ('3.14', ie 14 March in the US date system), on which mathematicians do inventively silly things based on bad puns with 'pie'. Make up any examples; the truth will be even worse.

e 2.71828182845..., going on forever and never repeating[42]. Named after either '(e)xponent' or '(E)uler', who first described it. Whichever people say it was, insist it was *actually* the other. If asked why e is important, waffle about '*basis of exponentiation... natural logarithms... compound interest... recurring in-joke in* The Simpsons*, because the scriptwriters were mathematicians... when Google first floated, their target value was $2,718,281,828, another in-joke*', etc. These last two can lead you nicely away from having to explain in detail what e actually is used for.

ϕ 'phi', pronounced 'fie', 1.61803390887[43]..., going on forever and never repeating. Often called 'the golden ratio'. ('A misnomer, used naively by non-mathematicians', you state.) The height-to-width ratio of an A4 (or A3, or A0, or any 'A' size) sheet of paper is ϕ, and it crops up in nature ('*leaf positions... snail shells... fast-breeder rabbits*', etc). But you can ridicule any attempts to give it mystical or aesthetic significance: '*tests show it's not the most aesthetically pleasing shape for rectangles, it doesn't remotely describe anything about navel height, and any use of it in art or music is just wishful thinking*', etc.

i Can't be written down any other way, except perhaps coming perpendicularly out of this page. It's the square root of minus 1, in other words 'imaginary'. (But say '*"imaginary" is a misnomer by Descartes, now used naively by non-mathematicians*' .)

If any genuine mathematicians talk about other 'constants' or numbers, especially ones described by a Greek letter, you can play for time by asking a few all-purpose questions ('*Is that transcendental? Is it normal? Has that been proved? But what about e to the power of it? Is that transcendental? Is that even irrational? Can we prove that?*', etc)

Infinity

Another good number to know, written '∞', although it's stretching a point infinitely far to call it a 'number'. It's more of a concept or ambition, really: what's the biggest number? Hah! One more than any number you can come up with! Actually, children have it about right.

Mathematicians love infinity, because it predicts lots of weird stuff. What happens when you do that infinitely many times? What about when that gets infinitely small?

Is the universe infinite? Maybe infinity is just something we've invented, and that the universe only goes so far. Or maybe it does go on forever – however, most people think there isn't enough room.

Infinitely long numbers Lots of numbers, such as π, go on forever if you try to write them out as a decimal: 3.141592653589793... etc. We can prove it never repeats, but we're not sure if *every* sequence you can think of will be in there if you wait long enough. If so – such a number is called a 'normal' number – then some weird things result. Such as, if you represent A as 01, B as 02 etc, you could eventually find your name, or a recipe for custard tarts in Polish, or your teenage diaries, or the works of Shakespeare, or anything, hidden in the long line of decimals. Someone will mention monkeys and typewriters here, so you can tut and smile and say that's a popular nonsense. (In 2003 Plymouth University gave six monkeys typewriters, but after a month all they had done was produce five pages consisting mostly of the letter 's', before defecating on the typewriters.) Mention (with the unspoken implication that you've read it) Jorge Luis Borges's short story *The Library of Babel*, and its library consisting of all possible 410-page books[44] (ie, all possible sequences of letters, most of them inevitably gibberish, though you can insert your own joke about books you hate being gibberish anyway). Maintain he clearly understood the whole maths/ infinity thing 'better than many mathematicians'.

The Hilbert Hotel A concept mathematicians like talking about, perhaps because it explains infinity quite well. Or perhaps because we rarely get to stay in big hotels, at least not on expenses. Imagined by the great German mathematician David Hilbert, it has an infinite number of rooms. Someone arrives looking for a room, but the hotel's full – so can we fit them in? Yes: the person in room 1 moves to room 2, the occupant of room 2 to room 3, and so on. Room 1 is now vacant, so we can fit them in there. (Obviously, there'd be a bit of a wait to change the key cards over.) Then a million people arrive looking for a room – can we fit *them* in? Yes: do a similar trick, but moving room 1 to room 1,000,001, room 2 to 1,000,002, and so on. Then an infinite number of people arrive – can we fit *them* in? Actually, yes: move room 1 to room 2, room 2 to room 4, room 3 to room 6 and so on, thus freeing up all the odd-numbered rooms – an infinite number. (Further trickery can cope with an infinite number of coaches turning up, each containing an infinite number of new guests, and even beyond, such as an infinite number of ferries each carrying an infinite number of coaches each with an infinite number of new guests, etc...) If the maths angle gets confusing, avoid it by joking about the practical consequences of such a large business – queues at checkouts for instance – though note that the hotel's presumably infinite car park could be the venue for another favourite type of maths problem setting[45].

Some infinities are bigger than others This fact is surprising to many non-mathematicians, so it's a good thing to talk about. The 'number' of positive whole numbers is infinite, but it's the same-size-infinity as the 'number' of all whole numbers (negative and positive), and the 'number' of all possible fractions. Whereas the 'number of all decimal numbers', while also being infinite, is a *bigger* infinity. The key difference is that you can write down all whole numbers in a sequence. Even – with a bit of ingenuity – all fractions. But you can't write down all *decimals* in a sequence, because between any two you can always slip another one in between. Usually, people will protest, 'But infinity is infinity! How can one be smaller than another!', and you can smile and sit back with your arms folded triumphantly, and direct them to web pages on Georg Cantor, the man whose fault all this is.

Adding up infinite sums Some sums might be infinitely long but have a finite result. Take a half; add a quarter; add an eighth; add a sixteenth, and so on forever. Visualising it as a cake, you can see that it gets as close to a full cake as you like. In other words, $^1/_2+^1/_4+^1/_8+^1/_{16}+$... and so on forever 'sort-of-equals 1'. Whereas $^1/_2+^1/_3+^1/_4+^1/_5+$... and so on 'sort-of-equals infinity'. With luck, talk of cakes will distract people so you don't have to prove any of this.

CHAPTER 5: MATHS IN EVERYDAY LIFE

Spot clever stuff in simple things, from washing lines to buses coming at once

Everything in life has some aspect of maths behind it. If you can point them out, it sounds like you understand how the world works better than everyone else.

Go to see the pyramids

Pyramid schemes are a common scam. To join the scheme, you contribute something – it could be money, or a book, or a pen-pal letter, to someone already on the scheme. In return, you receive similar from subsequent new members of the scheme.

The schemes promise huge returns (invest $1 and receive $1000, send one book and get 36 back, and soon) but only work for the first few members. That's for the simple reason that the number of new members mushrooms, but the population of the earth is very finite, so it soon runs out of people. By the time you hear about it, on Facebook or Instagram, it's too late: you'll send off your money or book and get nothing back, which is why pyramid schemes are usually illegal.

In Albania, many people lost their life savings in such schemes there in the 1990s: at one time it accounted for half the country's GDP[46].

In essence, any scheme that promises you integer-multiplied investments thanks to lots more people joining it after you is bound to be dodgy. (Recruitment schemes setting you up as an 'agent' who recruits more people for promised commission are invariably dodgy too, for the same reasons.)

Dismiss mention of such schemes wearily: '*Cynical traps for the naive and greedy... pity people unable to grasp exponential growth... we'd never get taken in of course... did you know "gullible" is not in the Oxford English Dictionary?*', etc.

Thrown under a bus

We all know you wait ages for a bus then two come at once[47]. Simple, you say: it's an example of 'bus bunching', a recognised phenomenon. (Slip in other terms, such as 'convoying', 'platooning', 'Poisson process', or 'remote feedback Linear-Quadratic Gaussian control schemes'.)

Typically, buses all start out at regular intervals. All things being equal, they'll stay the same distance apart. But things are not equal: a few seconds' delay to the front bus means they are delayed, pick up passengers who would otherwise have caught the bus behind, which delays them and speeds up the one behind even more. The feedback loop continues (maybe mention '*a* *butterfly beating its wings eventually causing a storm...*' here, though it's not relevant) and soon you have the two buses behind each other.

Imagine a line of sheep being herded up a Yorkshire Dales lane[48]. When they set off every tenth sheep is black, and the rest are white. Reacting to events en route, the sheep rearrange as they get herded along. When they arrive, the pattern will clearly be messed up, and you're impossibly unlikely to be back in the original state of every tenth sheep being black. They'll be clumped up at random, which means some black sheep will be bunched together.

In fact, this sheep thing has little to do with bus bunching and doesn't prove anything, but it's a good way to get to sleep.

Share and share unalike

Another place to demonstrate your maths knowledge is sharing cakes. Fairly dividing something nice to eat between two argumentative children is easy: one makes the division, the other chooses which 'half' to take. (Well, depending on the child, perhaps. 'Which half do you want, Milly?' –'I want Harry's'.) Abraham and Lot used this method in the book of Genesis, though for land, not cake[49].

But how to divide a cake 'fairly' – 'envy-free', to put it technically – between three or more argumentative children, or Biblical figures, or councillors, say? Maths comes to the rescue, though it's the sort of rescue that suggests it might have been easier to drown. For example, a 'cake' might be defined in one of the proposed sharing methods as 'C, a finite subset of the multidimensional Euclidean plane R^d', which isn't what you typically find in recipe books.

There are many ways to approach the problem, all complex. They have names like 'Last Diminisher', 'Fink Protocol' or, most excitingly, the 'Robertson–Webb rotating-knife procedure'. With squabbling children. What could possibly go wrong...

It's quicker just to bake more cake[50].

Look after Number 1

In any large table of figures that aren't restricted to some narrow range – lengths of European rivers, footballers' earnings, or simply 'all the numbers mentioned in today's newspaper' – you might expect that the first digit of each is equally likely to be 1 as 2, or 3, right up to 9. In fact it isn't. Almost a third of them will begin with a '1', about 18% with a '2', and so on, with under 5% beginning with a '9'. It's an effect independent of units (it doesn't matter whether your river lengths are in miles, metres or furlongs) and is usually called 'Benford's Law'.

If asked to explain why, say it's '*explained neatly on Numberphile's YouTube channel, and is really obvious once you see it*', then talk confidently about '*the chances of having a lottery winning ticket beginning with "1" when you don't know how many tickets have been sold*', in a manner that suggests this reveals the reasons clearly.

You can also maintain, truthfully, that it can be used in forensic accounting, to see when people have been making up figures in tax returns or expenses claims, say: they'll typically make them equally likely to begin with 1, 2, 3 etc, or overuse a favourite number such as 37, whereas genuine stats over a large enough range will have the uneven Benford distribution[51].

Forensic accountants have other non-Benford ways of spotting bogus stats, but are understandably reluctant to reveal them, perhaps because their own expenses claims might then come under scrutiny.

Compound interest: Avoid the scams

Financial trends over time: here's the place the mathematically naive are most likely to get fooled.

The trick is the compounding: there's interest on the interest, which builds up over time in a way that can be against expectations. (It's an example of exponential growth, though not dramatic enough to be noticed in the short term.) If rail fares are going up a barely-noticeable 3.5% a year then after 20 years prices haven't gone up by 70%, ie just over two-thirds, they've actually doubled (up 99%).

The '72 rule' helps you here: dividing the interest rate into 72 gives a rough estimate on the doubling time for a growth situation like this. If house prices rise at 7% a year, they'll approximately double after ten years, and so on. Conversely, if your savings are decreasing by 7% a year, they'll have about halved after a decade, making your house deposit four times as unaffordable. (You can continue with the dire implications for young people getting on the housing ladder, thus using maths to blame whoever you like.)

Some UK banks offer 0.01% interest on savings accounts, which – as the above rule shows – mean it would take around 7000[52] years for your pot to double in value.

The main thing to remember with loans is to look for the APR – the equivalent annual rate of interest. Often deals will quote the monthly repayment interest rate (such as on credit cards). But a manageable-sounding 5% a month (the figure they might quote at you) equates to 80% over a year: the APR (which may be hidden away in the small print).

Power to the people

The Pareto Principle, or the 80/20 rule, states that for many things, 80% of the effects come from 20% of the causes. For instance, the richest 20% of people earn 80% of the income, or 20% of the employees in a company do 80% of the work. (Except that *everyone* thinks they're in the 20%.) You can make out that this applies to any situation and will sound confident and clever.

It's an example of a power law: a relationship where doubling one figure means the other increases or decreases by some other, constant, factor. Moore's Law is a classic example of a power law – that computer power doubles every 18 months – '*though it's reached physical limits now*', you can say.

Zipf's Law is another example. This says that in any language, the frequency of a word is proportional to the rank – in other words, the most common word is twice as common as the second most common, three times as common as the third most common, etc (and the third most common is two-thirds as common as the second, and so on)[53]. Yes, it even works for swear words, so you can name-drop your favourite worst-area-of-town here.

Tenuous power 'laws' are quite fashionable now, and you can write books on economics based on them – for instance, '*doubling the company size only increases expenses by a factor of 1.7*', which '*explains the dominance of chains and multinationals*'. Make up your own power laws to 'explain' anything ('*Doubling what you pay for shoes gets you a pair that last 3 times/ only 1.5 times longer, so more expensive/ cheaper shoes are the better buy in the long run... IT'S A FACT*').

Pack animals

Packing problems are a fruitful area of maths. Literally, because the classic problem is, what's the most oranges you can stack on a greengrocer's table? (In which case they're actually "ORANGE'S"). In 1611, the astronomer and wannabe grocer Johannes Kepler conjectured that the most space-efficient way to pile them was the 'face-centred cube lattice', which greengrocers have always done anyway, though it wasn't until 1998 that it was proved. If you pack oranges like this in a crate and squash them together, each orange becomes a rhombic dodecahedron, from the Greek for 'unsellable'.

In two dimensions, can-packing problems clearly have industrial uses. It's easier to prove results for infinitely large boxes than finite ones, which is irritatingly what shops tend to work with. In the real world there are some surprisingly messy results. For instance, if you have a square box ten cans of beans wide, it can maximally hold not 100 but 105 cans – packed mostly hexagonally, honeycomb-style, but with a space in the middle and one can jiggling round in it.

Finding the best methods to pack randomly-sized things in identical boxes is a challenge that's as exercising to mathematicians as to people moving house. The rule 'take items at random, seal box when full, start next box' isn't too bad for efficiency: with 20 boxes' worth of randomly-sized stuff and 20 boxes, that method gets within 70% of the best fit, so at worst you'd need 34 boxes.

Streamline *that* method by packing the biggest items first, and you get within 22% of best fit, needing only 25 boxes at most. Then, once they're all closed and sealed, you realise you haven't labelled them.

Packing problems have applications far beyond everyday items. ('*Vital in computing and the internet, coding information in the most compact way... chemical and crystal structures that determine how creation works... structure of DNA... fundamental to life*', etc).

But I didn't vote for democracy!

Democracy is only economic-cake divison or power-cake division on a large scale, so it's no surprise that maths has things to say here too. Unfortunately, being maths, such things are largely incomprehensible to simplistic thinkers looking purely after their own interests, which means most prominent politicians.

The name here is Kenneth Arrow, a 20th-century mathematician and economist. He came up with reasonable-sounding conditions for a fair electoral system ('If every voter prefers candidate X to candidate Y, then X should prevail over Y', and so on). Then he proved that no possible electoral system could satisfy all the conditions: 'Arrow's Impossibility Theorem'.[54]

In other words, democracy never works. ('*We knew that already. But it's great to have mathematics confirm it. Or perhaps just depressing...*' etc)

Wonky chairs at wonky tables

If your four-legged cafe table is wonky, keep rotating it. If all legs are the same length, maths guarantees that there's an orientation somewhere that makes it steady, with all four legs in contact with the floor. There's an intuitive 'proof' that you can claim to understand. Consider three of the legs – they're a tripod which always have a stable position on the floor. Rotate the table through 360 degrees: some times the fourth leg must be above, and at other times notionally below, the floor level – so there must be at least one point in between where it was level with it. (Obviously, a similar idea applies to chairs and chair legs.)

Make some vague rotational hand gestures while explaining this, fingers spread imitating the table legs, and you'll be convincing. If it doesn't work in practice, blame the 'different length legs'. Note that the table top may not be horizontal, so ensure any spilled coffee dribbles away from you.

Formal proof didn't arrive until the 2000s, and there are some requirements – that the table is rectangular, the floor isn't ridiculously uneven, etc. But, in principle, there's bound to be a way of positioning the table so it's not wobbling, and your coffee won't spill any more. Unless you're with a particularly clumsy mathematician.

Roads to ruin

The Braess Paradox, named after the German mathematical modeller who described it, shows that adding a new road to a network can sometimes *increase* congestion – or, conversely, that removing one can decrease it. It's down to the way drivers change their optimum route to utilise the new option: individually it makes sense but the combined effect can lead to more jams.

Mention this in discussions of traffic flow to browbeat those tedious people who think they're experts on urban planning just because they have a car[55]. ('*Modelling accurately predicts how building roads to tackle congestion never works... Nash equilibrium may not equate with optimal network flow, politicians too dim to understand that... induced demand... gridlock... Mumbai, Bogota, Moscow at standstill...*', etc)

Realms of the coin

Two British coins – the 20p and 50p – are not round but curved polygons, with seven sides. It's because an odd-sided regular shape, with curved sides, can be made so that it always has the same width, whatever angle you rotate it to. So, vending machines can easily check it's the right width. (Bermuda had a triangular coin with the same property.)

The recent change of the pound coin's shape from round to 12 sides means it doesn't have the same consistency, because the trick doesn't work for even-sided polygons. The pound's width can vary between 23.03mm and 23.43mm – if somebody's coin gets stuck in a machine you can blame it on this. With old people you can recall the mid-20th century brass 3d piece, which also had 12 sides[56], and debate whether therefore everything was better then.

Fractals

Fractals are a class of self-similar shapes – complex curves-within-curves that have the property of looking very similar whether you're looking through a telescope or a microscope, such as lightning bolts, earthquakes or pineapples. The concept is illustrated by the joke about the French mathematician associated with them (said to have been devised by the man himself).

> Q: What does the 'B' in 'Benoit B. Mandelbrot' stand for?
> A: 'Benoit B. Mandelbrot'.

Fractal shapes have some mind-bending qualities, like being say in between 2 and 3 dimensions, and have exciting names like 'Sierpiński Sieve' or 'Menger Sponge' that sound like they'd be invaluable in an infinitely recursive kitchen.

The fractal thing is not quite as fashionable as it used to be, but you can still impress people by pointing to any shape that looks similar at different scales – coastlines, graphs of stock market movements, broccoli etc – and saying '*ah, classic fractal... nature's way of creating nested high complexity with simple, compact generators... chaos theory... our universe one atom in a universe of universes and so on, infinitely in either direction... secret of creation... does God play dice?*[57]', etc.

Pandemics

Some people think they're an expert on epidemiology because they once saw a graphic on Instagram. So there's no point challenging their views on, say, masks or social distancing. Instead, stick to discussing the maths of the situation – they won't be so sure of themselves there.

They're bound to have only a vague concept of the much-quoted 'R number', the 'reproduction number' the notional number of people that one person infects at any stage in the pandemic. An R of bigger than one is bad news – the number of infected people is increasing exponentially.

If pressed to explain exponential growth, waffle about how 'the rate of increase is proportional to the number', and mention compound interest, bacteria multiplying in a petri dish, and nuclear explosions, all with the ominous expression of someone who understands just how mathematically close we are to armagheddon.

In the unlikely event that they actually know what exponential growth is, pick holes in all of their assumptions. Do they know what R_0 ('R-nought', the 'basic reproduction number') is and why it's important, and why it's different from R ('R', the 'effective reprodction number')? How does their R-based model take account of death rates, immunity rates, vaccines, recovery rates and so on? You don't have to know the answers to these yourself, of course – a few key phrases stolen from academic papers plucked at random online will do nicely.

Stress that the maths itself is straightforward ('*such as differential equations...*') – it's the assumptions behind the model ('*how social distancing might affect R...*') that make all the difference. That will get people arguing about whether driving to go for a walk is irresponsible, and avoid you having to specify what a differential equation is.

A few facts, either committed to memory or made up, will make you sound like an expert: '*Of course, R isn't the only factor. Death rate matters. Measles has an R_0 of about 15 but rarely kills, so is less of a threat than say Ebola, whose fatality rate is over 50% for an R_0 of 1.5.*' Check the latest figures for fatality rate, and R_0, of any new diseases, and quote these sombrely, as if the implications are clear. Keep warning everyone that '*we don't yet have completely reliable data*' – this is always true.

Mug up on historical plagues from the internet, to prove that new ones aren't a patch on old ones, or vice-versa, but most importantly, to scare people off challenging your maths: '*The global* *pandemic in the year 540, the "Plague of Justinian", killed 1 in 5 of the world population... the Spanish invaders of Mexico brought diseases that killed 90% of the natives in fifty years... Spanish flu killed more people than World War I... bubonic plague... Ancient Egypt... dinosaurs... apocalypse*', etc.

Things of shape to come

The everyday world is full of shapes chosen for a particularly eye-catching mathematical quality, as the popularity of cosmetic surgical procedures shows.

Here's a list of some common shapes to look out for, what they're called by mathematicians, and why they're there. Only the naive talk about those slopes on which skateboarders do their astounding acrobatics as a 'half-pipe'; you know it's a brachistochrone, but also why it's that shape in the first place.

Brachistochrone Or 'curve of fastest descent': a downward slope (a type of cycloid) whose shape means a ball-bearing rolls down in the least time (yes, faster than a straight line or rounded L-shape). Used in half-pipes, ski-slopes etc. Also the curve which results in the fastest arguments, to judge by the spat between the Bernouilli brothers, who each claimed to have found it in the 1690s, and never went skiing together again.

Cycloid Curve traced by a point on a rim of a circular wheel as it rolls along. The Spirograph toy popular in the 1960s traced out variants of this, the hypotrochoid and epitrochoid, which you can use to claim that everything's been dumbed down since. Pops up all over mathematics but it's tricky to point out day-to-day examples, such as the pattern traced by the point on a rotating train wheel, as the train's probably stuck at Berwick because of signal failures. However, a favourite mind-boggler is to point out that however fast a car is driving, there's always a point on the tyre that's stationary (at the bottom of the cycloid). May not help you contest the findings of a speed camera, though[58].

Catenary The shape assumed by a sagging washing line without laundry. Upside-down catenaries are the most stable free-standing profile, so their arches were used inside Gaudí's Sagrada Familia in Barcelona, and Budapest's main railway station. Good places to mention casually, as it implies you're well-travelled.

Euler spiral or 'clothoid'. Its curvature changes evenly along its length, so it's the ideal transition shape between straight and regularly-curved sections of railway track. Blame your rickety UK train cornering not only on 'outdated rolling stock' but also 'the useless clothoids', which helpfully sounds like an insult.

Fibonacci spiral Pattern related to the Fibonacci series[59] that crops up all over the place: you can find on pineapples, sunflower seeds, Sanskrit poetry, hurricanes etc. You can point at random to any of these or other phenomena and say '*Look! 1, 1, 2, 3, 5, 8, 13 and so on... see? Classic Fibonacci!*', and people will be so busy losing count they won't challenge you.

Gyroid A triply periodic minimal surface, no less, discovered in 1970. Rather technical, molecular-level/ crystallography concept, but an impressive-sounding one to talk about. ('*Butterfly wings are so beautifully iridescent because the chitin that makes them up is shaped like gyroids... tremendously useful for making 3D printing work more efficiently... the reason ketchup sometimes doesn't come out of the bottle*', etc.)

Hyperboloid Cooling towers at power stations need to be curved to work well, but straight girders are easier to make than curved ones. Imagine two rings, one at the top and one at the bottom, joined by lots of straight girders. Give the top one a slight twist round. You end up with a cooling tower shape: a hyperboloid, a curved shape structured around straight lines. And arguments about fossil fuel emissions.

Minimal surface Something like coalescing soap bubbles, that contract so as to have the smallest overall surface area. The shape of the Munich Olympic stadium roof is an example. You can point to any old piece of modern architecture and say '*ah, that's a minimal surface, but maximum cost*'.

Parabola The shape assumed by a sagging washing line with laundry on it, or the cables of a suspension bridge. The three-dimensional version is used for the mirror in vehicle lights, and focuses the beam in a straight line forwards. Which doesn't explain why bicycle lights are so damn dazzling these days.

Platonic[60] solids The five regular shapes (ie all faces identical) that dice can be made out of. A conventional one is a cube with six sides and numbers 1–6. The other four shapes are the tetrahedron (four sides) octahedron (eight) dodecahedron (12) and icosahedron (20). Most modern footballs have 32 panels and result from inflating a 'truncated icosahedron', which has its corners lopped off to make 12 pentagons and 20 hexagons. ('*The same structure as buckminsterfullerene – C_{60} – and geodesic domes*' you can say, especially if you want to steer the conversation away from football.)

Wallpaper groups There are exactly 17 different ways of making wallpaper patterns repeat (a fact mathematicians only confirmed a century or so ago, which may explain why we prefer plain walls). No problem if you can't tell which is which; people will be happy that you're apparently complimenting them on their choice of decor. ('*Nice wallpaper! I see you've gone for p3m1 symmetry. So much classier than boring old p2 symmetry, I think*.')[61]

As a bonus, see if you can claim any of the above shapes '*also occur in share movements*' and '*are used in key algorithms that can predict the next stock market boom/ crash*'. Utter tosh, but many badly-informed people will believe that. Economists, for instance.

CHAPTER 7: PROBABILITY

Give yourself an even chance of probably beating the bookies 9 times out of 10

Open up any popular maths book. Chances are, they'll be talking about probability. Its main application is to gambling, or as it's sometimes called, 'life'.

At a basic level, probability is a simple concept: toss a coin over and over again, and you get heads half the time, tails the other half. Throw a dice, and there's a 1 in 6 chance of getting each number. And a fifty-fifty chance that some pedant will say you ought to have referred to it as a 'die'.

Mathematicians love pointing out examples of where our gut feeling about odds proves wildly inaccurate. Here's some standards you should know. It doesn't matter if you don't understand. If the person you're talking to doesn't have a clue, direct them to the Wikipedia page and YouTube videos for it. If you're talking to a mathematician, they'll excitedly give an explanation, which you can listen to and nod along with as if you knew all along what a Markov Chain is.

Football match birthdays Take two footy teams plus the referee – 23 people altogether. What's the chance some of them share a birthday? It's actually more than 50%. (NB If the people consist of ten sets of twins and one set of triplets, it's nearly 100%, but not quite – the twins might all have been born either side of midnight, and the triplets born on a plane crossing the international date line around midnight.)

Gambler's Fallacy For fair independent events – dice rolls, lotteries, coin tosses – previous results are no guide to future performance, like your pension fund. Five heads in a row doesn't make a tail next more likely: it's still 50–50[62]. That didn't stop players of the Venice lottery wasting money by piling cash on number 53 in 2004–05, thinking it was bound to come up because it hadn't been drawn for two years. Such a mistake is the 'gambler's fallacy'. Similarly, just because gamblers keep making this same mistake over and over again doesn't mean they're less likely to do so in future.

Lottery coincidences Your chances of winning are slim. In fact, pretty much whenever you buy a ticket, you're more likely to die before the draw. It would thus be a very unfortunate coincidence if you won and died next day, though it's arguable whether it would be 'ironic'[63]. Nevertheless, there are so many lotteries in the world, and so many things that might catch our eye, that eye-catching coincidences are bound to happen. On 1 Dec 2020 the South African lottery winning numbers were 5, 6, 7, 8, 9 and 10. On 21 Mar 2011 the Florida Lottery's winners were 14, 15, 16, 17 and 18. The German Lottery's winning numbers on 21 Jun 1995 were identical to those on 20 Dec 1986. The Bulgarian Lottery's nightly winning numbers of 6 Sep 2009 were repeated on 10 Sep 2009. Similarly, a couple from Scunthorpe won £1m on EuroMillions in 2013 and 2015; and in 2018 a man in Sydney, Australia, won £1m twice in a week. Nothing remotely suspicious about any of this: it's all in line with what theory predicts. (Sometimes though, it may be less of a coincidence: Joan Ginther won the Texas Lottery four times, pocketing $20m in total, thanks to buying lots of tickets at unusually favourable rollover times[64] – and having a maths PhD.)

Monty Hall Problem A mind-bending classic said to have originated in a US TV game show. There are three boxes, A, B and C. One of them contains a prize[65]. You make a guess. The host then opens one of the empty boxes – and then gives you the option to *change* your original guess. Is it worth changing? The gut feeling is that it makes no difference. But in fact, if you change, you'll win the prize two-thirds of the time. Sticking means you *lose* two-thirds of the time. Most people refuse to believe this – in which case you can win plenty of pretend money by doing a few dozen trials with them.

Boarding pass problem A nice one even lots of mathematicians won't know. The first person to board a full flight loses her boarding pass. The hurried cabin crew tell her to sit anywhere. They tell the other passengers as they each board to sit in their allotted seats, but if that happens to be taken, just to sit anywhere at random. When the last passenger boards, what's the chance that the only seat remaining is the right one? Surprisingly for most people, it's 50%. (There's a neatly intuitive way to see it by imagining that, as each new passenger boards, if they find someone sitting in their seat, the sitter gets up and sits in a random seat, and the new arrival gets to sit in their proper seat – you effectively have the same situation as if they hadn't swopped. The process continues until the last person arrives with one vacant seat left, and that first passenger sitting either in the right or the wrong seat, the chances being 50-50.) Coincidentally, that's about the same chances that the flight will arrive in time for your last train home.

Medical test fallacy You've been selected at random for a test on a disease that affects one in a million people. The test is 99% accurate for positive results (and never gives a false negative). The test comes up positive for you. What's the chances you have the disease? It's not 99%, but only 0.01%. All the test has done is make you 100 times more likely to actually have the disease than you were before the test. So instead of 1 in a million, it's 1 in 10,000. Conclusion? There's no point in testing people at random for diseases that are rarer than the accuracy of the test – you'll get too many false alarms. For similar reasons, a test that could (say) examine people's web browsing history and flag up if they might be terrorists would have to be more accurate than the incidence of terrorism, which is only one in millions. In other words, the test must be 99.9999% accurate or better – no chance of that. Make up stories about Facebook flagging you as an assassin because you bought a Ridley Crossbow (it's a bike) or a racist because you talked about Chinese pigs (in a post about swine flu affecting pork production).

How to beat the bookies

There's an obvious answer here. The Law of Large Numbers shows that the more you do something with fixed odds, the closer the odds work out. That's how a casino makes its money: so long as the odds are just in its favour, even 51–49, it'll win in the long run, even if a few individual punters might win in the short term. So maths proves that the best way to maximise your winnings is to not bet at all. That way you end up on the positive side of expectation 100% of the time.

Know what odds mean. '3 to 1' means the horse would win once for every three times it doesn't – in other words 1 in 4, or 25% chance of winning. If you total up the odds for all the horses in a race they should come to 100%, because the race will be won by one of them (except the 1993 Grand National, maybe). However, the total won't be – it'll be a bit less than 100%, and that difference is where the bookies make their money.

There can be loopholes sometimes, if different bookies offer different odds in a certain way. Suppose one bookie offers 5 to 4 for Cambridge winning the Boat Race, and another bookie offers 5 to 4 for Oxford. Bet £4 on each and (ignoring tax etc) you're bound to win £5 plus your £4 back, so a guaranteed profit of £1 – 'arbitrage', as it's called. The only problem is, this rarely happens. Bookies survey each other's odds and often prevent you from making such sneaky cross-bets, and you'd have to bet huge amounts to make it worthwhile.[66]

There isn't any mathematical 'system' guaranteed to make you money in any gambling situations. The nearest is the 'Kelly Criterion', which is more a way of optimising your strategy in a situation with some guaranteed winnings – say, the unlikely setup of being able to bet from a fixed pot of money on a coin toss that you know will land heads say 60% of the time. Kelly shows how to bet a fixed proportion of your bankroll so as to optimise those winnings in the very long term, and has applications in big financial investments. So, not yours.

Frequentist versus Bayesian

What is probability? What does it mean to say 'there's a 50-50 chance it'll rain tomorrow' – or indeed, 'there's a 50-50 chance it rained yesterday', when the event has already happened? There are different approaches, which affect how you use the mathematical tools available. (If pressed, claim that *'ultimately, probability is all philosophy, rather than maths – all maths does is supply the number-crunching tools.*')

The two approaches you need to know about are Frequentism and Bayesian Inference, and if you believe some of what you read in popular maths books, adherents of the two factions battle it out like rival sects of a schismatic fundamentalist religion, such as Economics.

Essentially, frequentists work out the chance of something by doing it lots of times and analysing the proportions of outcomes. Bayesians are more concerned with combining bits of knowledge to come up with a probability that sums up our beliefs of what's likely to have happened or is going to happen.

Take, for example, a fight between a Frequentist (F) and a Bayesian (B). But being mathematicians, they wouldn't fight, they'd play chess. So, what's the chances F will win? Or B? Or of a draw?

Frequentists would find out by staging, say, a thousand chess matches and coming up with figures to reflect their findings, justified with 'p-values' and 'sigmas'. You don't really need to know what these stats mean, but can look like you do by quibbling: '*Hmm, p=0.05? You think that's really small enough? Surely you need less than 0.01?*' Or: '*Only three sigmas? Shouldn't you play safe with four?*', etc.

Bayesians, however, would start off with a decent guess – F 40%, B 40%, draw 20%, say. Then, they'd use Bayes' Theorem to change the figures in the light of new information. F turns out to be a county-standard chess player with an 80% win record, for instance. But then they find out that B is a grandmaster with a 95% win record. Then the forecast predicts heavy rain, increasing the chance of a draw. The Bayes formula enables them to tweak the expected probabilities. ('*The formula itself is ludicrously simple*', you say, '*clear enough for a bright child of twelve*'. That should stop anyone challenging you, in case they come out looking dim.)

Frequentism is used for analysing experiments, such as when CERN found evidence of the Higgs Boson in 2012, with a significance of *five* sigmas, which is clearly enough, whatever that actually is. It's the basis for pretty much all medical trials, and you can snort at the lot of them. This is because they work on a 'null hypothesis' – the suspiciously vague notion of how unlikely the results are to have been a fluke, so they 'must have been due' to the effect being tested. You can rubbish this, and hence *all* medical trials: '*it's based on a lie, that the probability of B-given-A is the same as A-given-B. It's all a conspiracy by Big Pharma.*'

Bayesian Inference is used in all sorts of real-world situations, such as in-play gambling, where one side scoring a goal in a football match changes the odds of who's expected to win. (Whenever these in-play percentages change you can nod sagely and say, '*ah, Bayesian probability, of course*'.) Or, say, searches for a plane lost at sea, where new information about the currents or last known position of the plane can narrow down the search area, thanks to Bayes.

Or, indeed, working out from symptoms and tests whether the deceased really did have that disease.

You can maintain that '*Bayesian Inference ought to be used more in court cases too, working out probability of guilt when new evidence comes to light, for instance, but has been expressly ruled out by British judges as too complicated for juries to understand – it's a scandal*', and use any miscarriage of justice you can remember as 'evidence'.

And who was behind the formula? Thomas Bayes was an 18th-century English minister, but his formula wasn't published till after his death. Much is unknown about him, such as the year he was born, how he learned his maths, or what he looked like, so all we can come up with is a few best-guesses tweaked with other probabilities that we do know – which is rather appropriate.

Randomness

This is one of those quasi-philosophical questions. 'Random' means 'cannot be predicted'. But if you do enough trials – of, say, drawing lottery balls – maybe some unevenness in the output might appear. After a zillion trials, perhaps the smaller-numbered balls – having less paint on and therefore being lighter – might turn out to be *slightly more likely* to float to the top of the machines that select them, for example.

When your computer needs to generate a random number, it may well base it on something such as the 10th decimal place of seconds on the time-clock, which will be random enough, but not *technically* random, because it was determined by a repeatable procedure.

'*It's almost impossible in the real world to get genuine randomness*', you can confidently state, noting that 'in practice' a few techniques come close enough, such as 'Mersenne Twisters' or 'the Christopher Columbus generator'. Perhaps true randomness, you can say, will only come from 'quantum randomness'. That needs quantum computers to calculate it, which don't exist, so you're safe from having to explain anything there.

Statistics

A huge area of maths that often feels like a luxury-brand hotel in an earthquake zone: a lavish structure built on shaky ground that people use because their peers do. Statisticians are all well versed in bluffing, because passing off guesses as fact is what they do, so you have to proceed with caution.

Perhaps the best approach is to question their assumptions all the time. Sooner or later they'll have to admit they've made some arbitrary ones – a classic fudge being to treat any group of stats as a particularly handy sort of shape called the 'bell curve', aka 'normal distribution'. Such as with human heights, for example, even though they're clearly *not* normally distributed. That would require the possibility of someone being *any height at all*, 1729m, or 0.3mm, or -1.83m, which is clearly nonsense.

('*But why did you assume a normal distribution to work out the mean? It could be a Witch of Agnesi curve, and that doesn't even have a mean*', etc)

Correlation

How well two sequences of figures fit, suggesting that they might be linked, is a favourite way to manipulate truth. For instance, superimpose graphs of temperature and ice-cream sales, and they'll probably follow similar patterns: they correlate. This might support your theory that people buy more ice cream when it's hotter. Or perhaps that buying more ice cream raises the temperature.

There's a whole website (Spurious Correlations) devoted to finding such mathematically justifiable, but clearly nonsensical, links: margarine causes divorce, spelling competition wins mean death by spider bite, bee populations determine marriage rates, and so on.

So, 'correlation is not causation!' is something you can always say to challenge anyone proposing a link between things. Argue with all their assumptions, and insist that they're making unjustified connections: '*You claim that eating bacon is associated with increased incidence of depression. But maybe bacon eaters tend to have fried eggs with it, and they're what's actually responsible. Did you control for that? Or maybe depressed people tend to comfort-eat bacon butties, thanks to most clinics being near a branch of Greggs*[67]*. Or maybe vegetarians lie about being cheerful. Or...*' etc.

CHAPTER 11: THE MEDIA

They always get maths wrong. How to sound like you'd have been right

How can you tell when TV, radio or newspapers get their maths wrong? Simple: look for a sentence with a number in it.

The BBC radio series *More or Less* documents such media mishaps; claim to be familiar with it. Such checks-and-balances for dodgy stats are rare though: a 2007 study of 1,200 factual errors (not all maths ones) in US newspapers found that only 2% were ever corrected. The same researcher later found that 'self-evident' maths mistakes and 'unquestioning use of figures' were rife in news stories.

You may find this depressing. Or, if you work for a newspaper, uplifting. It means you can claim in print that 90% of news stories are correct, and everyone will then just copy it. You need never bother to correct it, and only one US researcher will ever notice.

Confusing 'millions' and 'billions' is a common gaffe (speculate on what sort of mistakes they make in hyperinflation economies such as Venezuela or Zimbabwe). As is saying things like 'the aircraft plummeted four or five hundred feet in as many seconds'. Percentages bamboozle a lot of hacks, too: '54% said Yes, 32% said No, and 26% said Don't Know', for instance.

Usually, though, it's not so much that they get the maths 'wrong' as they misuse it, to spin the story to have as eye-catching a figure, or headline, as possible. For example, 'drugs gang sent to prison for a total of 120 years' – it means little without knowing how big the gang.

Here's some more media maths fails to watch out for, and how to sound like you would have done it properly.

Unsourced data As the old joke goes, 99% of stats are just made up on the spot. Beware any figure that has no source quoted – it could well be a hurried journo's estimate. Cite the true story, from 1964, of the hack who needed to know how big the Sudanese army was for a topical story. Unable to find a source, even from Sudanese embassy, he guessed: 17,000. The story was picked up by news agencies, and after a few days, the Sudanese embassy rang him: 'We've finally found that figure for you. The Sudanese army is officially about 17,000.' (NB Thanks to the internet, things would be different now. It would only take hours to 'become fact'.)

Significant figures When converting figures, from mph to km/h say, you stick to the same number of 'significant figures' – the degree of rounding-off in the figure. A bike review talking about 'stability at speeds of around 20mph' should be translated as '...around 30km/h', not '...32km/h'. And certainly not '...32.1869km/h'. Make up more examples and claim to have seen them in print or on the BBC website: 'a miss is as good as 1.609343km', '28 grams of practice is worth 907kg of theory', 'You look like a million dollars (£821,870)', etc.

Relative risks A staple of good-for-you/ bad-for-you medical stories: not the risk itself, which might be tiny, but the change in the risk. A headline like 'wearing a cycle helmet boosts your chance of surviving a crash with a car by 20%', for example, might lead to you a different conclusion from 'your chances of dying in a high-speed collision with a car are 99% with a helmet, 99.2% without one', even though it's the same stat. Criticise any story that talks about percentage risk in this way, and if you want, you can always conclude that bacon/ coffee/ chocolate/ red wine etc is all fine, as they make no significant difference to your baseline chances of bad health. The correct way to put figures into context, you insist, is to talk about the extra number of people affected or not affected. ('*So, if half cyclists currently wear helmets, and 100 people die per year in high-speed collisions with cars, then if all of them wore helmets, the figure would be... 100. Hardly an argument for helmet use.*')

Original sin Any graph or bar chart that doesn't start at 'the origin', ie zero, is probably trying to exaggerate something. A measly rise of 1% in something, from 100 to 101, will be invisible if the axis starts at 0. But if the axis starts at 99, it'll look like it's doubled. Typically accompanied by words like 'soar' or 'plummet'.

Small sample size When a niche sport is described as 'the fastest-growing, doubling every year', it probably means that two people did it last year and now, thanks to corralling two friends, there's four. Which is clearly not as impressive as doubling from, say, two million to four million. The same goes for claims made in any study, survey, trial or press release unthinkingly copied out by an intern hack: if it doesn't apply to thousands of things, it's probably meaningless, because tiny random fluctuations radically affect the numbers. In 2019, for instance, York's papers reported the scandalous '33% annual rise' in bike theft. The actual figures were a nudge up from 9 to 12, at the railway station: still only one per month, from the thousands of bikes parked there daily. But '0.008 more bikes per day were nicked from York station last year' doesn't have the same ring.

Hotter at the double When warm spells result in a rise from say 10C to 20C, watch out for reports of 'temperatures doubling'. '*So*', you can comment snarkily, '*I suppose a rise from 1C to 10C is ten times as hot, and 0C to 2C means we've got infinitely warmer?*'.

Once-a-century flood How come 'once-a-century' floods are apparently so frequent? Even if the maths is simple, the implications can be involved. A place that on average floods once every hundred years has a near-two-thirds chance of flooding in any given century period. And the actual situation is never that simple. Records rarely stretch back centuries, so that '1-in-100' figure is at best restrospective guesswork – and depends on odds not changing over time, which they clearly do, thanks to climate change, alterations in land use and so on. Then add the fact that different places flood differently, and a headline-seeking sub-editor will happily misapply a '1-in-100' figure from one part of a river to another part ('which might be a 1-in-10-year area'). So you can ridicule any 'once-a-century' (or 'one-in-a-thousand', or 'month's rainfall in a day' etc) news item for being wrong in theory and in real life. As with other maths issues, look up the Wikipedia page, pick out impressive-sounding phrases, and insert them into your conversation at random (‘*stationarity... recurrence interval... distribution function...* ’). Then slag off insurance companies, because everyone will join in with *that* conversation, and their own tales of outrage.

Which average? New stories are always talking about average this and average that, but rarely say which one. There are three common ways of calculating ‘average’: mean, median and mode. The ‘mean’ is everything added up and divided by how many there are; median is the middle term, if you put them all in order; mode is the most common. Each may give different answers, and you can always sound clever by quibbling over which one they’ve used and saying they should have used another. For instance, ‘*The mean UK income is £34,200, but the median is £28,400, according to the Office for National Statistics (ONS). Newspapers choose which one according to whether they’re pro- or anti-government.*’ Apart from earnings, the other thing everyone in the UK wants to know is the ‘average house price’. You can sound confident and clever here, because they use another type of mean – not the more usual arithmetic mean as above, but the geometric mean[68]: ‘*The ONS reckons the average house price in England is £256,000. Obviously that’s based on the geometric mean from 2015 with a hedonic regression model applied. And in any case it’s meaningless because high/ low house prices in London/ Hull distort everything.*’

Bogus formulas A relatively recent development: adverts masquerading as press releases masquerading as a ‘formula’. Typically the first paragraph will read ‘Boffins from [obscure university with name like a metro station] have found the formula for the perfect Christmas tree/ pop song/ penalty/ sandwich/ humorous maths book’ etc. Unlike real mathematical equations, which have superscripts and Greek letters and integral signs and so on, these suspiciously simple impostors only have plus or minus signs and upper-case letters of the normal alphabet. The usefulness of such a formula is defined by the formula

$T = C + 2M - G$

where $T = 0$.

CHAPTER 13: TRICKS

Impress your friends. Annoy your enemies

Coining it in

Line up two £2 coins side-by-side, with the queen's heads both facing due west. Now roll the left coin clockwise 180 degrees round the right one, keeping the edges in contact, so the coins end up side-by-side again. How many revolutions will the coin make? The intuitive answer is 'half', but it's actually a whole one: the queen ends up facing due west again. Obviously this works with any other value coin, but as the other person will be too distracted trying to work out what's happened to ask for their coins back, you want to maximise your takings.

Bear essentials

You walk a mile south, then a mile east, then a mile north – and find yourself back where you started. What colour is the bear you can see? The well-known answer is 'white', because 'you must be at the north pole', as it's 'the only place' you can do that triangular walk and end up where you started. Actually, though, there's an *infinite* number of other places where you can do this, all of them just above the *south* pole. (If you start at any point 1.159ish miles north of the south pole, then just after you've walked a mile south, your walk of a mile east is a circuit exactly once round the south pole, so walking a mile north takes you back to where you started. There are even more possibilities based on your circumpolar leg being two, three or indeed any number of circuits.) So, presumably, the bear is a mislaid cuddly toy belonging to one of the Antarctic research staff, in which case it could be any colour.[69]

Welcome to the fold

Take a normal sheet of A4 paper[70]. It'll probably be 80gsm, which has a thickness of 0.065mm. Fold it in half: it's now clearly twice as thick, so, 0.13mm. If you fold it another 20 times, how thick will the resulting wad of paper be? The result is not 2.6mm, but over 136m (because you multiply it not by 20, but by twenty lots of twos multiplied together, which make just over a million). In fact, you'd only have to fold your A4 sheet 43 times to make it stack up higher than the moon. And a mere 73 folds would pile it up beyond the edge of the observable universe, 46 billion light-years away. Obviously it's hard to fold paper that tightly. A bit of origami skill may be handy here. If you can make the paper into a crane, say, you may be able to deflect questions about exponentiation, or how many kilometres a light-year is.

Not what you think!

This neat trick-question, which did the social media rounds in 2019–20, has the pleasing effect of making normal people think they're right when they're wrong, and mathematicians think you're wrong when you're right. It asks you to solve this sum:

230–220x0.5 =?

and then quips, 'You probably won't believe it, but the answer is 5!'. Now, most people never paid attention in maths lessons, and solve it from left to right: 230 minus 220 is 10, multiply by 0.5, that's 5, right? They can't see the problem. Anyone who did pay attention in maths lessons knows how to solve it correctly, however: multiplication comes first, so you do the 220x0.5, get 110, subtract it from the 230, and get 120. So surely the question is wrong because the setter knows nothing about maths? Nope: the catch – as you know all along, of course – is the exclamation mark. Because 5! denotes 5 factorial, in other words 5x4x3x2x1, which is, of course... 120[71].

Interesting times

A neat trick to talk about is 'x% of y is y% of x' – a handy way of making calculations with percentages an easier way round. If you need 16% of 25, for instance, that's the same as 25% of 16, in other words, 4. Except that, in the real world, the figures never fall so conveniently: 17.5% of 189 is no easier than 189% of 17.5. (Though, to be fair, it makes it easy to see that the answer is 'a bit less than twice 17.5, ie a bit less than 35'.) However, it's neat not because you'll ever use it, but because it impresses people who'd never thought of it. (Be nonchalant: '*it follows from multiplication being commutative, obviously*'.) They might say things like, 'Why didn't they teach me this at school?', to which you can respond, '*They probably did, but you weren't listening*', because they won't be listening.

Cube routes

Solving a Rubik's cube isn't easy, but there's a sneaky way to make it look as though you are doing so. Essentially, any repeated action (swizzle top layer once, turn 90 degrees clockwise, for instance) will eventually cancel itself out. So, take a solved cube, do such a repeated action a few times to scramble it, and put the cube aside. When people enter, nonchalantly pick up the cube and continue your action until the cube returns to the solved state. Matt Parker's YouTube video shows you how to do it. There are plenty of other tricks on his channel too – so note the URL, and make sure you never tell your friends about it.

Multiple issues

To really bewilder someone, try this multiplication trick. Describe it as a method 'known to the ancients since Stonehenge' or some such tosh. (It's sometimes called the 'Russian peasant algorithm', for no good reason, and so you can make any irrelevant point you like about *that*.)

Take any two numbers – 37 and 19, say. Your mobile phone's multiplication app will come up with the answer in an instant: 703. But your aim isn't to get the answer – it's to bamboozle the other person.

So, write two columns with 37 at the top of one and 19 at the top of the other. Successively halve figures in the left column and double them on the right, ignoring remainders. You get:

37	19
18	38
9	76
4	152
2	304
1	608

Say 'even numbers have to be removed because ancients believed they were evil' or '...female' or some such nonsense, and cross them out from the left column (so, lose the 18, 4 and 2) along with the partner numbers in the right (so, lose the 38, 152 and 304).

Now add up the remaining numbers in the right column: 19 + 76 + 608, which gives you... 703.

Don't attempt to explain why it works (and it always does). Just smile knowingly, perhaps mumble about 'binary numbers' and 'distributive property of basic arithmetic' – but certainly say '*it's obvious once you see it*' and '*funnily enough, kids get it straight away*', neither of which is remotely true, but it'll get the other people worrying that they're going to be replaced by someone younger and better-looking.

Venn and now

Everyone will be familiar with Venn diagrams. Intersecting three circles to show what sets have in common, or not, is easy. (Sets A, B and C need areas for 'none', A, B, C, AB, AC, BC, and ABC – eight possibilities altogether.)

But how can you do it with four sets, ABCD, say? Trying circles won't quite give you all options (only 12 of them, in fact). But you can cover all 16 possibilities (none, A, B, C, D, AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD, ABCD) with ellipses. (Or indeed two circles and two kidney shapes, or lots of sausage shapes, though you may need a trigger warning for vegans.) Recommend that someone does this, and that they fill in the sections with all different colours, so they 'can see *why* it works'. As they hunt for enough differently coloured pens, you can make your excuses and leave, without having to come up with a real-life example of when you might *actually need* four-set Venn diagrams[72].

Episode of Simpson's

'Simpson's Paradox' is a trick where – if the numbers line up in a certain way – a trend common over two sets of data *reverses* when you combine them. The most famous real-life example was when the University of Berkeley, California was criticised for bias against women in 1973. Overall admission rates for men were far more favourable than for women. However, department by department, the *opposite* was true: men had worse admissions rates. Everyone thought they were being discriminated against, and everyone was unhappy except the lawyers.

The effect is best demonstrated by an example. This one makes out politicians to be hapless, forced into making bad decisions whether they like it or not, which you can use to sympathise or attack them, as you like. But you can adapt it for any other group to prove that we're all hapless slaves to statistics.

Two rival train companies, A and B, are bidding for a new franchise. The minister is legally bound to award it to the company with 'the better record on punctuality'. The figures come in:

Rush hour	**A**	**B**
Trains	500	300
No. late	80	51
% late	16%	17%

Off peak	**A**	**B**
Trains	100	500
No. late	10	60
% late	10%	12%

A is more punctual in both periods, so they get the franchise.

But then company B's lawyers furiously re-present the same figures as totals, which prove that *they* run fewer late trains overall:

Totals	**A**	**B**
Trains	600	800
No. late	90	111
% late	15%	13.875%

Company B sues the minister, the minister resigns, the government topples, the railway system is renationalised instead, and everyone is happy. (Adapt this ending as you like.)

Watch and smile as people work through these figures in disbelief, convinced there must be a trick somewhere. There isn't, but you can confuse them further by talking about '*confounding*' or '*lurking variables*', *'positive and negative correlations... back-door criteria in decision-making... graph theory... you can prove anything with statistics...'* etc.

The infinitely puzzling sum

What do you get if you add up all the whole numbers? In other words, what's 1+2+3+4+... and so on, going on forever? The short, obvious answer is 'don't be silly, it's infinitely big', and basically, that's right.

However, with some frankly dodgy manoeuvring, though kind-of allowed by stretching a point, you can contrive an answer of $^{-1}/_{12}$. Yes, 'minus a twelfth'. The great mathematician Ramanujan thought this was fair enough, and there's even a real-world phenomenon in physics called the Casimir Effect where this is consistent with what you observe in the lab.

Mathematicians tend to roll their eyes at the '$^{-1}/_{12}$' result as a blatant fix ('a complete misuse of the axiom of choice', etc), just as taxpayers roll their eyes when a hugely profitable multinational company's accounts claim tax liabilities of two quid.

You probably should too, though you could try a loftier view: that it shows how nuanced the idea of 'rules' is, how elusive the idea of 'truth' versus 'opinion' etc. Particularly if you're an accountant working for a hugely profitable multinational.

The Banach-Tarski Paradox

Possibly even stranger than the 'infinite sum' above, the Banach-Tarski shows that with a finite number of cuts, you can chop up a sphere into bits that reassemble into *two spheres of the original size*. It's tricky in practice, because the cuts are infinitely detailed, so it wouldn't work with chocolate oranges.[73] Raise philosophical questions here: '*It makes us ask what precisely we mean by "volume". What we mean by "cut". What we mean by "by"...*', etc.

The unbelievable blocks

This is an incredible consequence of the fact mentioned in the 'Numbers' chapter that if you add enough of the fractions $1+{}^1/_2+{}^1/_3+{}^1/_4+{}^1/_5+$... etc, there's no limit – you can make the total as big as you like. For this trick you'll need a lot of same-sized slabs of some kind, such as dominoes, or multiple copies of the same unsold paperback book, breeze blocks, gravestones etc.

Pile up the blocks, flat, on top of each other. Then nudge them sideways so that each higher one juts out a bit more than the the one below. You have a skewed stack, with the skew getting gradually more as you go up, not quite enough to make the stack fall over. (There's no cement; they stack under their own weight.)

At the bottom they're almost aligned but at the top they can jut out further – in fact the very top block can overhang the one below by half its length before everything topples. The second-top block can overhang the one below it by a quarter; the third-top, a sixth; the fourth-top, an eighth; the fifth-top, a tenth; and so on.

The question to ask your sceptical observer is, What's the maximum overhang of the top block with the respect to the bottom block without falling over?

Astoundingly, the answer is 'as far as you like'. That's because adding $1+{}^1/_4+{}^1/_6+{}^1/_8+{}^1/_{10}+$... is clearly half as much as adding $1+{}^1/_2+{}^1/_3+{}^1/_4+{}^1/_5+$... etc, which we know can be as large as you like. So half of as much as you like is still as much as you like. You just have to keep adding far enough down the line, but as the desired total gets bigger, you have to go unimaginably further down the sequence.

For the top block to be able to overhang the bottom completely you only need a stack of four. To overhang by two blocks' distance, the stack is 31 high. For three blocks' worth, it's 227 high, for four it's 1,674... while an overhang of 15 would need six trillion blocks piled up, and an overhang of 100 would require a pile with far more blocks than there are atoms in the universe.

This can be fun to try with dominoes, but a standard set of double-six only has 28 tiles. Still, you may win more money betting that you can stack them to overhang by nearly two blocks width than you would playing dominoes.

CHAPTER 17: ALGORITHMS

Facebook, Google, price-setters, facial recognition, self-driving cars... there's less to them than it seems

Everyone's heard of algorithms, but few people have much clue what they are. They may even be afraid of them. So you can exploit this. '*Ha!*', you laugh. '*Being afraid of an algorithm? That's like being afraid of a recipe!*'.

The first thing they'll ask you is to explain what one is. Try the old engineer's joke as an example of an algorithm: '*If it moves and it shouldn't, apply duct tape. If it doesn't move and it should, use WD-40*'.

An algorithm is a series of instructions to solve or find something. It might be as vague as the above, or as detailed as a computer program. For instance, 'an algorithm' for finding your way to the seaside might be this: *Head north until you find the coast (if you hit a river, follow it until it reaches the coast). Then turn right and head along the coast till you find a beach. (If you get back to where you first hit the coast, there's no beach.)* It'll just about work, but the fact that it just about works is no guarantee of efficiency. Even Google Maps will be quicker.

Algorithms supply the structure for computer programs, and computer programs control a lot of our lives these days. That's because of the vast amounts of data about us – personal details; history of internet banking, web browsing and purchases – most of which we've given up ourselves by agreeing to those unread 300-page terms and conditions. It's the data that's the problem: it allows people to write algorithms to read it and make decisions based on how they process it. Which Facebook posts we see; whether our e-passport photo matches our face at Immigration; which offences the government might trump up against you based on your secretly reported mobile phone location data, such as proximity to protests about state surveillance.

Take a dating website, for instance. It takes the data you supply – your hobbies, interests, favourite music, background and so on – and a computer program compares it to other people's, in a process shaped by the algorithm. Maybe it calculates some sort of 'close fit' between your educational backgrounds, or your musical tastes, and decides which factors are more or less important. In an ideal world, the algorithm would match you with your partner. You're already matched. Or perhaps not, seeing as you're both furtively using the site.

Or your email software's spam filter. It cleverly analyses keywords found in your incoming emails, and intelligently processes them against a comprehensive, recently-updated list of known scams. Then it puts the one about buying Bitcoins full of links such as 'install-spyware.exe' in your Inbox, and sends that job offer you were waiting for to the Junk folder.

‘Algorithms’ are to maths what ‘recipes’ are to cookery. If the sponge cake failed, it’s because of something the recipe didn’t mention (eg not opening the oven door too soon). Or the cook didn’t follow the recipe (eg substituting granulated for caster sugar). Or the recipe itself was wrong (eg too much cooking time). Whatever the case, you don’t have to know any *details* of the recipe: simply blame ‘the recipe’. Similarly, for computer fails, you need know nothing about programming or maths: blame ‘the algorithm’.

(Note that the above is itself an algorithm.)

You can justify any opinion in this simple way. Why the self-driving cars will never work, for instance. ‘The algorithm’ for interpreting other road user’s behaviour will always fail to cope with something (ignoring a red signal to allow an ambulance through, for instance). ‘The algorithm’ for deciding a safe speed gets overridden manually by the driver. ‘The algorithm’ for distinguishing a pedestrian from a shadow simply gets it wrong.

(People love to talk about self-driving vehicles, because most people think they’re an above average driver[74]. Intimidate them with statements such as, ‘Level 4, ‘hands-off’, autonomy only works with geofencing – clearly impossible in the UK – so Level 5 is never going to happen’. Sooner or later, someone will raise the ‘Trolley Problem’: if a self-driving car suddenly has to choose between two options, one which will kill the driver, the other five pedestrians, which should it select? Tut and dismiss it as irrelevant, simplistic pseudo-philosophical nonsense – ‘*that’s to misunderstand what algorithms are*’ etc.)

Origins

The first major algorithm was Euclid’s, in about 300BC, for finding the highest common factor of two numbers[75].

But the word ‘algorithm’ derives from the last name of our old friend al-Khwārizmī, the brilliant and pioneering 9th-century Persian scholar from what is now Uzbekistan, whose books set out the structure of modern, equation-solving, algebra. He devised an algorithm for solving quadratic equations and is a good topic: you can steer away from anything that might show your ignorance about computing theory, or quadratic equations, by talking cheap, easy imponderables.

(For instance, ‘*Being a devout Muslim, he would be delighted/ amazed/ horrified that algorithms in his name are now used for determining the best route to Mecca/ diagnosing illnesses better than doctors/ working out how much to panic you into buying by nudging up the price of your flight since you looked two minutes ago*’.)

Types of algorithm

There are very many books on algorithms, some of which appear to have been written by one. Not so Hannah Fry's, *Hello World,* which talks about four main categories of algorithm:

Prioritisation: making a list Google's infamous algorithm ('PageRank', named not for web pages, but for Google co-founder Larry Page) to order your search results is the most common example. Precisely how it works is a secret known to few and understood by fewer, which means you can claim pretty much anything. ('Results are easy/ impossible to manipulate by metadata/ hiding keywords invisibly in the text/ clickfarms/ right-wing extremists/ left-wing extremists'.)

Classification: picking a category Handwriting decoders are an example, deciding which word or postcode that scrawl best represents, which is why your Christmas cards get lost in the post. Others tell advertisers what sort of person you are based on your data, and hence what you might buy: an expensive road bike for an affluent male turning 50, for example. Or a slot on a site for selling unwanted goods, for the same person a month later.

Association: finding links Amazon's recommendation engine is an example, looking at what past customers went on to buy after ordering something, in order to suggest stuff that you might go on to buy, too. (A balaclava after buying a baseball bat, for example.)

Filtering: isolating the important Speech recognition algorithms do this, such as bots which answer your phone call to the bank or the utility company. They separate out words from the background noise and make them into intelligible sentences. Just not the ones you actually uttered.

There are also two general ways in which algorithms work:

Rule-based Constructed by humans in a clear way. Simplistic, like the engineers duct tape/ WD-40 example above, but often very powerful and useful, and easy to modify because it works so transparently. Well, transparently to the programmers. Obviously they keep the workings secret to everyone else – making their algorithm a so-called 'black box'. This is a *bad thing*.

Machine-learning This is the way that computers mimic sophisticated human behaviour with highly variable, complex inputs and relatively simple outputs, where the rules are too complicated to work out explicitly. It's typically the product of 'neural networks', which 'mimic the brain' and 'learn' in a way 'similar to humans' without any specific 'rules' but by repeated tests with a 'goal' and 'rewards' when they get near the 'goal'. You'll be doing a lot of air-apostrophes with your index fingers when you 'explain' this.

Machine-learning works in a similar way to how you train a child to recognise a cat from a dog: point out lots of cats and dogs, and reward them when they identify correctly. Eventually, as if by magic, they work out how to process various layers of information – pointy ears, long muzzle, wagging tail etc – and sum them up with the simple cat/ dog result.

Neural networks

Neural networks do this by filtering data through many levels of simple yes-no classifications, and after enough time, they'll find their own way of working out which patterns mean a cat and which a dog. The problem is that it's totally opaque *how* they do this, and when they get it wrong, it can be wrong in a totally non-intuitive way. Misidentifying a bus as an ostrich, for instance, or trying to drive off a bridge to find the road, because all the roads it has seen so far have had grass verges at the side (both real examples).

To sound like you've read a book, or been to a talk, on algorithms and their pervasive effect on everyday life, and implications for the future, name-check any media mathematician, or failing that, '*researchers in California*': '*She shows that face-recognition tech simply isn't accurate enough... He demonstrates that Facebook's metrics are a load of tosh... They say fully autonomous cars are five years away, but they said that five years ago...*'

Why algorithms are a bit rubbish

A safe opinion on pretty much any involvement of 'algorithms' is that the danger is not in the computers' decisions themselves – they can often be useful, assisting doctors making a tricky diagnosis with many conflicting symptoms, for instance. It's that naive humans trust them too much. The problems start where good old common sense ends, you confidently declare[76]. Here are some examples, with useful opinions to hold.

Self-driving cars Will never be reliable enough for full autonomy. Humans must always be on hand, awake, alert, sober, to take over in the inevitable case of something the autopilot hasn't seen before – hence defeating the point. Only people who want them are car makers and their cronies in government.

Justice Already, algorithms are assisting police and judiciary in identifying suspects and deciding guilt. Slippery slope down to surveillance state. Chinese already snooping on citizens via facial recognition. Smartphones spying on us with covertly installed software. Talk about '*that case in Russia/ America/ China*' where authorities used bogus algorithm to send dissident to jail for life.

Understanding language Computers are great at literal translations or set phrases, but terrible at anything that demands everyday experience. For example, what does the 'it' refer to in this sentence? 'I tried to put my bike in the Azuma train's storage space, but it was far too small'[77]. To a human it's obvious: we know what bikes are, we know what storage spaces are, and we know you can't put storage spaces inside bikes. But a computer will have no idea, and won't understand that sentence. Mind you, neither did the people who designed Azuma trains.

Facial recognition Recent Google embarrassments (eg algorithm catastrophically misidentifying online pictures of people of colour as 'gorillas') only tip of iceberg. Though a computer can't tell an iceberg from a meringue. Facial recognition (eg for CCTV to scan crowds and automatically trace a suspect's movements) simply too complex for machines, so inaccurate it will waste more police time than it will save.

Autopilots Cite crashes, ideally with flight numbers for maximum effect, as classic examples of too much reliance on algorithms, and not enough manual control: 'Air France Flight 447... Lion Air Flight 610... Malaysia Airlines MH370', etc.

Here's a real example of what happens when algorithms go unsupervised: in 2011, two booksellers on Amazon advertised a book on flies – a work much in demand in academic circles, out of print since 1992. Both sellers had their price set by an algorithm, based on the others' price. Unfortunately the two algorithms kept systematically trying to outdo each other, mechanically multiplying the other's selling price by 1.270589. At one point the book was on sale for $23,698,655.93 (plus $3.99 shipping). Conclude, dramatically, with a shrug: '*And now, pricing, selling and buying algorithms control the world's stock markets. Financial armageddon is only a matter of time...*'.

Modern algorithms to know about

Monte Carlo Method (1946) Running squillions of computer simulations to find the most probable outcomes. Very useful in situations where no simple deterministic model exists – weather forecasting, for example, which depends heavily on big computing power. The UK's Met Office say their four-day forecasts are now as accurate as their one-day forecasts were in 1980, though the BBC switched to a cheaper supplier for their forecasts in 2018. Everyone likes talking about the weather, in particular how useless and inaccurate the BBC forecasts are, so you can use this to prove any point you like about supercomputers, snowflakes[78], or Michael Fish[79].

Quicksort (1959) Sorting things into order is a staple computer task, and Tony Hoare's algorithm is still one of the quickest and most convenient ways to do it. If asked to explain how it works, waffle about '*divide and conquer*' and '*space-optimised binary tree sort*', and stress its practical superiority over other sorting algorithms either real (heapsort, bubblesort, pancake sort, bucket sort etc) or made up (anything+'sort' is plausible).

Shor's Algorithm (1994) Process for factorising numbers quickly, thus making all internet security useless, and so breaking the world economy. Fortunately it only runs on quantum computers, which nobody knows how to make, because we need quantum computers to design them.

Fast Fourier Transform (1965) Cooley and Tukey's ingenious way of deconstructing a wave into its elements, such as sound waves into their component frequencies. Sound sampling works thanks to this algorithm. Gauss actually got there first, but never got the credit, because he preferred Beethoven to trip-hop.

Simplex algorithm (1947) Devised by George Dantzig as a quick way to find a near-best answer where many different things have to be balanced – originally the optimum blend of four aviation fuels – this is one of the most widely used algorithms in the modern industrial and technical world. Fund managers balancing investments, car manufacturers assembling their thousands of parts in the most efficient way and so on, all use it. Talk airily about 'polytopes': '*they're the multi-dimensional notional shapes describing the various possibilities whose edges reveal the best combinations... the Simplex provides a fast way of finding close-enough solution points... only a few hundred steps to solve 50-dimensional problems, instead of billions for a brute-force approach*', etc. The Simplex outperforms other methods 90% of the time. The other 10% of the time is basically insoluble problems such as working out the best domestic energy supply contract.

CHAPTER 101: SCHOOL

The syllabus hasn't changed for decades. So we know all the answers by now

Maths, the most hated subject?[80]

Rubbish, you can say. According to one recent poll, maths is British schoolkids' favourite. It was the top choice of 26% of the 777 quizzed (compared with 18% for PE and 14% for English) who are presumably also the ones good enough at maths to be able to tell you how insignificant and misleading such polls are (although others have found similar results).

Because maths is true, it's pretty much the same in schools today as it was forty years ago. Thanks to generations of teachers being pressurised into coaching kids in how to pass exams, pass rates have gone steadily up, which older people can use as evidence that either standards have increased, or everything has been dumbed down.

The only maths you'll ever need to use in real life has all been learned by the age of about 12. After that it's stuff you only need to understand vaguely, and once you get to university, you only have to admire YouTube videos about it.

There is, of course, a very reliable, accessible and cheap fount of knowledge for bluffing techniques: people who have learned how to get through the education system by the seat of their pants on a mixture of minimal maths knowledge and maximum pretence, bluster, rule-bending and luck-pushing, and who will happily show you how to do the same.

Your maths teachers.

A feel for numbers

Something that sounds like common sense, though anyone who doesn't have it can claim it's all rather unnatural and a human construct. ('*Past say four or five, animals only have a sense of rough size. Even the most intelligent collie has no idea of quadratic equations, and they do their job OK...*').

Essentially, this is just knowing the times tables, plus a bit of market-stall common sense: 19% of 502 is roughly a fifth of 500, so it should be about 100, so even if someone shows you '953.8' on a calculator as 'proof', you know they've probably keyed in something wrong. (Here's where everyone, including you, will join in with apocrypha about long-departed relatives: '*My grandad ran a shop in the East End, and he could tell you what 17 lots of three shillings and ninepence was in a flash... kids with calculators today have NO IDEA*', etc.)

Proportionality

Many maths teachers, even ones at university (cue more anecdotes about dumbed-down maths requirements to fill up places) bemoan some students' lack of sense of proportionality.

Some consequences of things move in the expected proportion: if you double the price of something, you double the VAT.

But some aren't. A 1-in-10-scale model car won't weight a tenth as much as the real thing, but a thousandth. And a car hitting you at 60km/h won't impact you with three times the energy of one going 20km/h, but nine times as much.

In other words, being hit by real car at 60km/h is equivalent to *nine thousand* model cars colliding with you.

An earthquake of magnitude 2 will shake your house not twice, but ten times as much as a 1-tremor. And one of 8 is not 'twice as powerful' as one of 4 – it's 10,000 times as strong. (Slap down anyone who calls it 'Richter scale' – 'that's the press being clueless, it's actually *moment magnitude*').

Similarly, a noise of 110 decibels doesn't pummel your ears 10% more than 100 decibels, but twice as much. So two motorbikes are as deafening as one live rock concert, and 32 motorbikes will equal an eardrum-rupturing 150dB, about the level you have to shout to order a round in a student bar.

And in real life, proportionality works in strange ways. Someone who earns a hundred times as much as you won't pay a hundred times as much income tax, but probably none at all.

Understanding the question

Most of GCSE and A level maths is simply a matter of reading the question. For example, suppose the examiner wants you to add two and two. They won't just say 'What's 2+2?'. They'll say something like, 'Consider two equal even integers greater than zero. What is the smallest possible sum of the pair?' The difficulty isn't the maths itself – it's just understanding what they're on about.

As a rule of thumb, the longer the preamble to a question, the simpler it is to answer, and the less foreknowledge you need to understand it – that's why it's long, because it's explaining stuff from scratch. Long questions are good options for haphazard revisers.

Multiple choice

The same tactic for all multiple-choice questions applies to maths. Go through answering all the questions you think you know. Then go back through the ones you don't know, applying a few rules: Longer answers are more likely than short ones; 'All of the above' or 'none of the above' are more likely than specific answers; and, Avoid consecutive answers of the same letter.

For the remainder, choose a letter you like and answer them *all* that way.

All that isn't guaranteed to maximise your chances, but it'll make you look confident and quick in the exam room, and might just spook everyone else into panicking and doing worse than you.

CHAPTER 1729: UNIVERSITY

Why maths is the easiest and best option for students

Search the internet for reasons to choose maths at university and you'll find some pretty guarded statements. Unis know that they can't oversell it, because the mathematicians will find them out.

Common reasons to do maths

Employment prospects Earnings for maths graduates generally are in the top ten or so subjects, with careers such as accountancy, actuarial work or – for those who can be bothered to read past 'A' in the careers handbook – financial wizardry, opening many doors. Often the sort beginning with 'trap-'. You'll always have options: there's a dire shortage of maths teachers, for example, so you can walk into any teaching job. Before running out of it the following year, when you find out why there's a dire shortage.

Transferable skills Mathematicians have a reputation for logical thought, unbiased approach, and the ability to champion the best solution to a problem rather than simply 'agreeing with the boss' or 'following the crowd'. Skills you'll obviously have to suppress in your first office job.

The world needs mathematicians The Enlightenment of the 18th and 19th centuries owed a lot to mathematics as it sought to move away from the old world of tyrannical leaders and a credulous, superstitious populace to a fair, just society based on education, understanding, logical thought and rational decision-making. One day we might even start the process.

Better reasons to do maths

Flexibility Not quite sure what you want to do in future? A joint course could be the answer. Maths is like a big bag of chips: you may not be able to finish it all, but it goes with anything. So you might be able to switch from full-on Maths to say Maths and Spanish, or Maths and History, or Maths and Mickey Mouse Studies. (Try passing that one off as real – '*a business studies module on Disney at London South Bank University*', that sort of thing. Most people will believe that.)

Free time Maths is the least time-consuming discipline, if you can get reasonably good at it: no long research hours spent in the lab or the library. You may well spend plenty of time thinking through those problems you've been set, but this can take place in your room, or the sports field, or the bar. You're actually working hard. So you claim, anyway.

No essays No worries about whether you've misused or misspelt that word. You'll use very few words in your written work: only words such as 'therefore', 'however', etc, and – when you suspect something but can't prove it – 'obviously'.

Other skills Universities are now fully aware that you need all sorts of skills to complement your problem-solving prowess. You may well get courses in PowerPoint, video or audio editing for podcasts, etc. When you come to do your first presentation and find your USB stick will only work on a PC but there's only a Mac, your problem-solving abilities will be tested to the full.

Fun Universities are desperate to attract mathematicians, and make great efforts to include plain entertainment in the degree, passed off as vital activities to enhance your understanding. If they don't offer these, suggest them. Fluid mechanics? Run across a vat of custard. Probability? Day at the races. Group symmetries? Learn juggling. Classical dynamics? Learn unicycling. Game theory? Play games, etc.

Working together

In decades past, doing a maths degree was a kind of sentence of solitary confinement. Not now: the scope of maths has mushroomed across all disciplines, and it is now massively collaborative. Which means you need have no qualms about asking for help. This isn't 'copying', it's 'networking'. Being able to come up consistently with the right answers without knowing what you're actually doing is a valuable skill.

It obviously won't be like that in the world of work. There, you'll only be expected to come up with *any old* answers without knowing what you're actually doing

Lectures

The lectures most worth attending are the ones just before the exam. The lecturers inevitably realise, in panic, that they've forgotten to cover a question which is about to come up. So watch for the telltale signs: 'I know I said we'd do an introduction to Galois Theory, but instead I'd like to talk about functions that are continuous but not differentiable everywhere. Here's an interesting example, called the Takagi Curve, or the Blancmange Function[81]…'

That means there's a question on continuous-but-not-differentiable functions coming up in the exam, and you're just about to get the answer provided on a plate. Particularly if it's the Blancmange Function.

Emergency techniques

There will be occasions where your proof to a problem isn't stacking up. Here's a few sneaky tricks to cover the joins.

The page turn Very often you know what you have to prove, and know what you have to prove it from. But having worked down from the question and up from the answer, you have a gap in the reasoning that you can't quite fill. Sometimes you can get away with some hand-waving text ('*continuing in this fashion, it can be seen that...*' etc). If not, the best idea is to contrive a page-turn between the steps. Most busy markers will read right past this one.

Proof by assumption Proof by induction is a powerful technique familiar to A-level students. You assume something is true for n, and show that if so, it's also true for n+1. Then you show it's true for n=0 – and hey presto, it must therefore be true for all n. Well, proof by assumption is a similar idea: you assume it's true for n, and show that if so, it's also true for n. Obviously you haven't actually proved anything, but wrap it up well enough and you might get away with it.

The magic wand Sometimes in order to prove things you need preliminary things that you know, or suspect are true, or at least it would be convenient if they were true, but you can't prove *them*. Brazen it out: simply quote the things you need, along with a confident '*By continuity/ symmetry/ the axiom of choice/ the authority vested in me, we can see that...*'

Doing a PhD ('Doctorate') or post-doc

You will get wearily used to answering the question, 'What is your PhD about, then?', with the title of your thesis receiving a bewildered silence and killing the dinner-party conversation stone dead ('It's about quasi-local Penrose mass for asymptotically anti-de Sitter space-times[82]... er, more wine, anyone?'). Post-doctoral research is remote territory, where the title produces a bewildered silence even from PhD students, and kills even faculty water-cooler conversations stone dead.

CHAPTER 10^{100}: MATHS STARS

How to sound like you influence the influencers

The concept of 'a mathematician' is fairly recent. Until the 1700s they were called something else, such as philosopher, geometer or astronomer. In the last decades of the 20th century we got called something else too, though not always as polite.

But an even more recent concept – so new it doesn't even have a proper role name – is the *media mathematician*: someone not primarily, or at least not exclusively, known for theorems or papers or setting reading lists to be ignored by undergrads, but for their ability to explain maths to a general audience through TV, radio and internet work. Or at least to think up jokes about partial differential equations.

These are good people to know about. Partly because they're good at making complicated stuff simple – the 'great maths teacher' you wish you'd had[83]. But mainly because – even if people don't quite follow that lively and humorous YouTube explanation of the Fourier Transform – everybody wants to have an opinion on their hairstyle or lack of one, dress sense, accent, line in gags about partial differential equations, etc.

So here's some. It's useful if you're producing a radio or TV programme where your presenters and star guests may need help understanding simple maths, otherwise running the risk of looking dim. Every radio or TV programme, in other words.

Remember, you only have to follow someone on Twitter to refer to them by their first name in conversation with others, as if they were personal friends: '*Hannah was on telly again the other day. Rather good, I thought. Seen Matt's new video? Really funny. Alex tells me he's doing a new book...*' etc.

Martin Gardner The late American science writer whose puzzles in popular magazines and books of the second half of the 20th century pioneered 'recreational mathematics' – the fun stuff, in other words. It's conjectured that all modern mathematical autobiographies start with the author being inspired as a child by Martin Gardner. No counter-example has yet been found.

Carol Vorderman Mental-arithmetic whiz on the UK Channel 4 game show *Countdown* whose ability to manipulate huge numbers came in handy for her tax returns – she was notorious at one point for appearing simultaneously on every terrestrial British TV channel.

Rachel Riley Vorderman's replacement on *Countdown*, considered the sharper arithmetist, and hence the connoisseur's preference. Give withering stare to anyone who criticises her dress sense: '*Well, surely it's her skill at the job that matters, not whether or not you like her frock?*', etc.

Rob Eastaway Author of popular maths books for all the family with a strong emphasis on understanding everyday life, with titles such as *How to remember almost everything ever* and *Why do buses come in threes?*. Gives talks, shows and lectures and sets puzzles for *New Scientist* magazine.

Often covers the maths behind sport, so is clearly a man who can't just explain what football managers mean by 'giving 110%', he does it too.

Matt Parker Stand-up maths comedian, writer, educator and YouTube presenter with a nice line in creative and humorous explanations of maths concepts, such as what shape footballs actually are, how to win at Monopoly, or who made the world's first alcohol-induced maths mistake. Has no fears of the aggressive, vicious and threatening comments that plague anybody with a YouTube presence – he survived being a maths teacher in Australia, so he can survive *anything*.

Hannah Fry British TV and radio favourite: a friendly face to explain with a smile the maths behind real-life situations such as shopping, dating, riots, disease, war, global destruction etc. Excellent role model for young women attracted by modelling. ('*No, mathematical modelling*', you can say caustically to anyone who misunderstands, implying they are out-of-touch and sexist. '*Women are allowed to be mathematicians now, you know. And it's Professor Fry, by the way...*', etc.)

Marcus du Sautoy The Simonyi Professor for the Public Understanding of Science who's the go-to telly and radio expert for high-level maths stuff. Producers: if you need someone to explain the Riemann Hypothesis or Poincaré Conjecture in the same lively and simple way that, say, Alan Shearer can tell you where Arsenal's defence is going wrong, Marcus is your man. And much cheaper. And he can probably explain where Arsenal's defence is going wrong better, too.

Ian Stewart Professor at the high-status maths department of Warwick Uni, and prolific author of readable, authoritative, enlightening popular-maths books. An expert in catastrophe theory, so has a good insight into the book publishing industry. You can cite him for anything and probably be right: '*Yes, I read about that in one of Ian's books. Good exposition, I thought, though of course he had to keep it simple for a popular audience...*'

Simon Singh Author of engaging, original popular-maths books, on subjects as diverse as Fermat's Last Theorem, cryptography, the origin of the universe, and maths jokes in *The Simpsons*[84]. Also wrote a book on the evidence for the effectiveness of alternative medicine, but resisted the temptation to make it 200 blank pages.

Alex Bellos Another author of engaging, original popular-maths books, and puzzlemeister for the *Guardian*'s fortnightly maths column. That's a goldmine for sounding clever, with often intriguing challenges about everyday-life situations that look tough to solve until some smart intuitive hack is revealed. Though of course you don't reveal that to other people. You let them think you worked it out yourself, whereas you actually waited for Alex to post the answers.

If the above sound a bit too mainstream, here's some 'expert's experts' names to drop.

Sir Tim Gowers Cambridge prof and pioneer of collaborative projects, for which his work on combinatorics may come in useful for predicting personality clashes
Martin Hairer Australian expert on stochastic analysis and Mac software developer, so he's used to working in abstruse systems
Karen Uhlenbeck US prof and self-described 'messy thinker', on messes such as geometric analysis and topological quantum field theory
Nira Chamberlain British data scientist of Jamaican heritage who models real-world problems, such as convincing industrialists that mathematicians are cheaper and more effective for saving money than trial-and-error

CHAPTER g_{64}: HALL OF FAME

Classic theorems of the greats, and how to look like you understand them

You don't need to know much about the history of maths – how anyone did long division in Roman numerals, for instance, or the mystery of how the Greeks managed debts without negative numbers. (Or, indeed, the mystery of how they manage debts now.) Non-mathematicians can take it or leave it, while with mathematicians it's the other way round.

However, it's good to sound familiar with some of the landmark people and theorems, and the people behind them. Here's all you need know.

The greats of history

The Babylonians Roughly 2000BCE–500BCE. They counted in 60s rather than 10s, giving us our second-minute-hour system and 360 degrees in a circle. Good knowledge of a lot of basic maths, such as quadratic equations, compound interest and loan sharking. Knew about the 3-4-5 right angled triangle, but there's no documentary evidence to suggest that builders used ropes to mark them out. We do know, however, that they blamed everything on the previous builders.

Pythagoras 'of Samos' (ancient Greece). We know next to nothing about him – the first mention of him was in the 3rd century, 800 years after he died – and there's no firm evidence whatever to link him with 'Pythagoras's Theorem' (that for a right-angled triangle, $a^2+b^2=c^2$, for example $3^2+4^2=5^2$.)[85] Any colourful stories you read about him (he led a cult, was vegetarian, died protecting some beans etc) are undoubtedly made up. So there's nothing to stop you making up a few more.

Euclid Lived and worked around 300BCE in Alexandria, though we know little about him either. Famous for his *Elements*, a book on geometry and number theory that remained a 'set book' for many centuries and is still important today[86]. Much of its content came from earlier authors, and Euclid isn't even mentioned as author or editor in early editions of the book. Hence, it laid the groundwork for the basis behind modern maths: axioms; proofs; logic; and, most usefully, the phenomenon of having other people's work passed off as your own.

Brahmagupta Seventh-century Indian mathematician and astronomer who made a huge step forwards, and backwards: he was the first to set down the rules for using zero and negative numbers, and hence inventing non-profit organisations and overdrafts. There's a spiritual angle to his writing which you can riff on ('*only in India, the land of Buddha, with their philosophical meditations on nothingness versus existence, and ideas of infinite rebirth, could they have considered "zero" as a meaningful concept, or considered valid the absence of a thing rather than a thing, or...*', etc).

Al-Khwarizmi Muḥammad ibn Mūsā al-Khwārizmī worked in Persia in the early 800s. Remember three things about his great and highly influential work, the *Compendious Book on Calculation by Completion and Balancing*. First, he pioneered the idea of algorithms – general procedures to solve problems – which are named after him ('al-Khwārizmī' became 'al-gorithm'). Second, it set out the rules for manipulating equations, laying the foundations for all modern algebra. Third, the world 'algebra' comes from the book's alternative, shorter title: *Al-jabr*, Arabic for 'completion' – which goes to show that, sometimes, it really is worth listening to the marketing department when they suggest something snappier than you came up with.

Cardano Girolamo Cardano (1501–1576) was an Italian Renaissance man whose talents included mathematics, especially algebra: he worked out how to solve cubic equations, possibly by stealing the answer from someone else, pioneered the use of negative numbers, and even investigated imaginary numbers. He also blazed the trail for probability theory, which was just as well – he was a keen gambler and always short of money.

Fermat Frenchman Pierre de Fermat (1607–1665) did outstanding things in calculus, number theory and probability, but is admired as maths' greatest bluffer thanks to 'Fermat's Last Theorem'. Since antiquity, we've known that you can find whole numbers that fit the pattern $a^2+b^2=c^2$ – for example $3^2+4^2=5^2$. But can you find similar solutions to $a^3+b^3=c^3$, or higher numbers? No, said Fermat, and claimed – in a note he wrote in the margin of a book – to have found a 'marvellous proof' of this, but that 'the margin is not big enough to contain it'. He was obviously bluffing, though the theorem was eventually proved by Andrew Wiles[87] (see below).

Newton and **Leibniz** Isaac Newton (1642–1727) of England and Gottfried Leibniz (1646–1716) of Germany were not only the independent co-discoverers of integral calculus, but have both have biscuits in their name (Leibniz-Keks and Fig Newtons – which were evidently named after a US town rather than Isaac, but it's a nice myth to perpetuate). The controversy over who got to integrals and differentials first was long and bitter, with accusations of foul play on both sides; in any case, it was Leibniz's notation that proved more convenient, and which is used today. No doubt about which is the more interesting character to bluff about though: Newton essentially did come up with his revolutionary theory of gravitation after seeing an apple drop from a tree[88], formulated the binomial theorem, split sunlight into the colours of the spectrum, visited pubs disguised as a tramp to investigate counterfeit coinage, practised alchemy, and died a virgin. Leibniz got the better biscuits, though.

Euler Swiss genius Leonhard Euler (1707–1783) is arguably 'the greatest' mathematician, whose long and productive career carried on even through blindness. Anything in maths that contains e, π, Σ, Π, logarithms, 'f(x)', or anything long enough to run on to the next page, and it's probably Leonhard you have to blame. He came up with the most famous single equation in maths: $e^{i\pi}+1=0$. In fact, whatever mathematicians are talking about, you can say, '*didn't Euler anticipate some of those results in his later work?*', and probably be right.

Gauss Carl Friedrich Gauss (1777–1855), from a poor German working-class background, was a child prodigy. He came up with deep stuff across many areas of maths, most memorably his Prime Number Theorem, which pioneered one of maths' most useful trends: naming theorems after what they're actually about. It's tricky to bluff about his work (unless you're a maths undergrad happy talking about Fast Fourier Transforms, say). Much easier to stick to making up or embroidering stories about his school days. For instance, '*When a teacher told him to add up all numbers from 1 to 100 to keep him quiet for an hour, the toddler Gauss instantly replied, "5050!" (because he spotted that 100+1 = 101, 99+2=101, 98+3=101 etc, until 50+51=101: so just multiply 101 by 50)*'.

The modern names to know

Sir Andrew Wiles (1953–) The English number theorist who in the mid-1990s proved one of the great unsolved questions of maths, Fermat's Last Theorem, after years of working in secret in his spare time. The Oxford Maths Institute's home is named the Andrew Wiles Building, and Sir Andrew has an office there. Make up stories of how any package meant for the building itself (stationery, furniture, catering packs of cauliflower etc) and addressed 'Andrew Wiles, Oxford' get delivered to him.

Grigori Perelman (1966–) The Russian enigma who resolved another long-standing mystery by proving the Poincaré Conjecture in the mid-2000s. He was awarded the Fields Medal but turned it down, as well as $1m prize money from the Clay Foundation (see Great Unsolved Problems). If you're unsure of what the Poincaré Conjecture is, exactly, talk instead about Perelman's mysterious and reclusive nature, exaggerating a few internet rumours: '*he's given up maths entirely in order to study mushrooms*', or '*he's now working in secret on the Navier-Stokes problem while working as a street cleaner in St Petersburg*', perhaps.

Terence Tao (1975–) Chinese-Australian now working in Los Angeles who proved an astounding result about prime numbers in 2004. Now, 3, 7 and 11 are three equally spaced primes (four apart); Tao proved that if you look along the list of primes far enough, you can any length sequence of equally-spaced primes. He works in many areas of maths, so if someone's talking about a subject you're not familiar with, you can say '*isn't Terence Tao doing something on that?*', and probably be right. And if you're not, at least you'll worry the other person. Particularly if they're researching that area and were hoping to publish on it first.

Cédric Villani (1973–) French star immediately recognisable for his 19th-century-dandy look (floppy hair, foppish cravat, spider brooch etc) and his work on partial differential equations (best stick to talking about the floppy hair, foppish cravat, spider brooch etc). Became a member of the French parliament in 2017 for a left-right-centre party – perhaps it takes an expert in Riemannian surfaces like Villani to understand the convoluted topology of French politics.

Many other mathematicians' names will crop up, but you can usually bluster or play for time by asking some red-herring questions ('*What country did they come from, again?*'; '*What was that theorem they did – no, not that theorem, the other, lesser-known one?*'; '*But that result was actually known by Euler, wasn't it?*' etc).

Here's a few quick categories of mathematician that might crop up.

Female mathematicians

Women didn't get the chance to do much maths before the 20th century, except for the famous conjecture – as yet unproven, but widely believed to be true – that it's all men's fault. Cite some or all of the following as proof that women have been outrageously either overlooked, or overrated, depending on how brave you're feeling.

Hypatia (Greece, 4th CE): Battled prejudice all her life. Better mathematician than the men of her day, it's said, which may explain why she was murdered. Died tragically young.

Sophie Germain (France, 1776–1831) Battled prejudice all her life. Brilliant algebraist and number theorist, but had to publish under a male pseudonym. Gauss supported her and recommended she be awarded a degree, but it never happened. Died tragically young.

Ada Lovelace (England, 1815–1852): Battled prejudice all her life. The world's first computer programmer, and the real brains behind Babbage's proto-computers, but never got recognition until afterwards. Died tragically young.

Emmy Noether (1882–1935) Battled prejudice all her life. Algebra genius whose work on symmetries shows why the laws of physics apply to the real world, thus proving that physics is all ultimately maths anyway. Hilbert supported her, but despite his recommendations she never got paid for her lectures or professorship. Died tragically young.

As for sexism and gender bias in the modern mathematical world, and whether it's steadily being overcome or continuing to be ignored, you can do what everyone in the world does: extrapolate from the incomplete data of your own experiences, real or imagined.

You'll need to know a few female mathematicians of the modern era. A name to drop is the US's **Gladys West** (1930–), whose work on modelling the shape of the earth helped the development of modern GPS, but who was late being added to Wikipedia, along with many other female scientists. You can safely rail against this fact.

Another is the Iranian **Maryam Mirzakhani** (1977–2017), the first woman to win the Fields Medal. Her work included topics such as 'counting simple closed geodesics on hyperbolic Riemann surfaces' – a counter-example to anyone who reckons 'women can't read maps because they can't visualise three dimensions'.

Tragic geniuses

The trope of the tortured, Romantic prodigy who dies young has (perhaps only) one big example in mathematics: **Evariste Galois** (1811–1832), the French political and pure-maths radical who died in a duel just as he was beginning to open up new areas of pure maths. The 'Galois Theory' is the idea that he wrote up everything he knew the night before his duel, and was so tired he couldn't shoot straight[89]. Rubbish such notions as exaggerated nonsense propagated in fanciful, casually-researched old popular maths books, such as the first edition of *Bluffer's Guide to Maths*.

Famous people who did maths

There are not actually that many of these – no UK Chancellors of the Exchequer, for instance, which may explain why the economy is so chaotic[90]. But if required, pick out a few of the following. If you like them, cite the fact they did maths as proof of their successes. If not, use it as proof of their failures.

Napoleon France's future emperor was adept at maths and retained a lifelong interest in it. There are so many myths about him that you can make up a few more maths-based ones (perhaps speculations on why it was not Josephine's night) and nobody will be able to dispute them.

Florence Nightingale The founder of modern nursing was also an innovative statistician, pioneering techniques such as pie charts and graphs to present the figures to support her work. If you've ever sat through a dire PowerPoint presentation, you know who to blame.

Trotsky aka Lev Bronstein, the Russian revolutionary who gave up studying maths to bring socialism to the world instead. Proof that mathematicians are deluded idealists or fearless truth-tellers, as you wish.

Lewis Carroll aka Charles Lutwidge Dodgson, the Oxford maths lecturer who wrote *Alice's Adventures in Wonderland* and *Alice Through the Looking-Glass*. Evidence either that mathematicians are experts in conceptual play, or that they live in a world of their own.

Arthur C Clarke The British science fiction writer took a first in maths and physics from King's College, London. Lived the last fifty years of his life in Sri Lanka, because of their more relaxed attitude to his lifestyle: mathematicians still endured public disapproval in the west at the time.

Tom Lehrer The brilliant American satirical songwriter enjoyed success in the 1950s and 1960s while lecturing in maths on the side. Afterwards he stayed in academia, teaching both maths and musical theatre. His take on academic plagiarism, *Lobachevsky*, was the first notable comic song to include the line ‘analytical algebraic topology of [a] locally Euclidean metrisation of [an] infinitely differentiable Riemannian manifold’. And the last.

Harvey Milk US gay rights politician who majored in mathematics at teacher training college – clearly a man not afraid of a challenge.

Art Garfunkel American singer who taught maths at a private school in Connecticut in 1971, at the height of his fame with Paul Simon. If the kids were expecting a rendition of Bridge Over Troubled Water, they were disappointed: they got double geometry instead.

Brian May Studied maths before graduating in Physics from University College, London, and then threw away a promising career by playing guitar with a local band (called ‘Queen’) before seeing sense and returning to finish his PhD (in astronomy) in 2007.

Dave Gorman British comedy writer and performer who dropped out of a maths degree at Manchester. Author of *Too Much Information*, so that’s probably enough about him.

Dara O’Briain Irish comedy writer and performer who completed a degree partly in maths at University College, Dublin. Fluent in Irish as well as maths, and said by his Wikipedia page to speak to his father exclusively in Irish. It doesn’t say if his father understands Irish.

Sergey Brin Russian-American who graduated in maths and computer science aged 19, but left his subsequent PhD to start a small business with fellow dropout Larry Page using some of the dusty academic maths nonsense they had been working on. The small business is now known as Google.

CHAPTER ∞: GREAT UNSOLVED PROBLEMS

You could win $1m (really). Here's how

In May 2000, the USA's Clay Mathematical Institute – set up by US financier and maths fan Landon Clay – published a list of seven major unsolved problems, offering $1m for a proof for each one. Only one has been solved so far.

Here they are in rough order of bluffing difficulty, easiest first, with all you need know:

– The problem's name, and its area of maths;
– roughly what it's about (resisting the temptation to say 'about 500 pages');
– a formal statement (ideally memorised for maximum effect, especially if preceded with a 'well, basically...', and illustrated with some diagrams in fact adapted from Ikea bookcase assembly instructions);
– a ludicrously simplistic description (best said with a straight face);
– the apocalyptic consequences if it were proved, which become vaguer as the conjectures get more complicated.

1. P vs NP: Computing

The question is... Are some problems, such as working out the most efficient route to visit a long list of cities, too time-consuming for a computer, or have programmers simply missed the quick ways to do it because they're too busy snowboarding?

Formal statement 'Is every language accepted by some nondeterministic algorithm in polynomial time also accepted by some (deterministic) algorithm in polynomial time?'

Ludicrously simplistic description 'Some problems are easy to solve and check on a computer, like multiplication. Some are tricky to solve and tricky to check, like working out the best move in chess. But some are tricky to solve yet easy to check, like sudoku. By "easy"we mean "P" ("polynomial"), and by "tricky" we mean "NP" ("nondeterministic polynomial"): because as you scale up such problems, it soon becomes impossible to solve before the end of time, a 1000x1000 sudoku for instance. But are they only tricky because we've not found a simple way to do it yet? That is, are all NP problems actually P?'

Consequences if proved '*Everything becomes solvable by computer. Internet security is blown away. The world becomes 100% efficient. Computers write the best novels and symphonies. Humans become redundant*.'

2. Poincaré Conjecture: Topology *Solved!*

The question is... If a four-dimensional duck walks like a four-dimensional sphere, swims like a four-dimensional sphere and quacks like a four-dimensional sphere, does that mean it *is* a four-dimensional sphere?

Formal statement 'Is every simply-connected closed 3-manifold homeomorphic to the 3-sphere?'

Ludicrously simplistic description 'Take a bunch of ants who wonder whether they're living on a grapefruit or a doughnut. The ones who live on the grapefruit note that if some of them join hands – OK, antennae – and form a circle, they can always move towards each other to make the circle smaller and eventually all bump into each other. But that's not true for the ones on the doughnut – if their initial circle goes round the doughnut's hole, for instance; then they can't shrink their circle tighter than the width of the hole. So a grapefruit is different to a doughnut. Is the idea. Except in four dimensions. And no, it's not important they're ants. They could be termites, or anything tiny. Look, ignore the ant thing, OK?'

Consequences if proved Grigori Perelman, the reclusive Russian researcher, cracked this one in 2004. He was awarded the Fields Medal for it, but turned it (and Clay's $1m) down. '*Which proves the trope of the lone-maths-genius still survives. And that some people still do things for pure motives, not just for the cash*.'

3. Navier-Stokes Problem: Fluid Mechanics

The question is... How come planes can fly?

Formal statement 'Given an initial velocity field, prove there exists a vector velocity and a scalar pressure field, both smooth and globally defined, that solve the Navier–Stokes equations.'

Ludicrously simplistic description 'The set of four differential equations of Claude-Louis Navier and George Stokes very usefully describes how viscous fluid flows. Custard, for instance. Or the air that keeps up the wings of a 737. Or that doesn't. But there are some unknowables in the equations, and sometimes they generate wrong solutions, or none at all, contrary to experience. The aim is to fix the equations so they model reality more accurately.'

Consequences if proved '*More efficient planes... better understanding of ocean currents... more accurate weather forecasts... room-temperature superconductors giving unlimited free electricity... smoother custard...*'

4. Riemann Hypothesis: Number Theory

The question is... Is there a hidden pattern to the prime numbers?

Formal statement 'Prove that all the nontrivial zeroes of the Riemann zeta function have real part ½'

Ludicrously simplistic description 'Numbers like 7, 13 or 2027 are prime because no number divides into them nicely. But we can't see the pattern. Solving Riemann's would reveal it, and unlock huge exciting new areas of maths, physics, finance and large-scale embezzlement.'

Consequences if proved '*Internet security, which depends on the pattern of primes being unknown, would be blown away, so fraud, hacking and state-sponsored e-terrorism would be rife. On the plus side, it might help us understand Dirichlet L-functions. So, it's swings and roundabouts.*'

5. Yang-Mills Mass Gap Theorem: Quantum Physics

The question is... Why are electrons so damn fat?

Formal statement 'For any compact simple gauge group G, prove that a non-trivial quantum Yang–Mills theory exists on R^4 and has a mass gap $\Delta > 0$'

Ludicrously simplistic description 'Physicists have spent years, and billions of pounds of other people's money, finding the 17 elementary particles that make up everything, like electrons and muons and gluons, the most recent being the Higgs Boson. Well, Chen Nin Yang and Robert Mills's theory neatly describes how all this works, but with a big problem: the theory says the particles don't weigh anything, but experiments show they certainly do. We need to fix the theory to explain that "mass gap".'

Consequences if proved '*At last, particle physics would make mathematical sense, and we could breathe easy, knowing that the universe works after all. Like French philosophers say, "that's all very in practice, but how does it work in theory?".*'

6. Birch / Swinnerton-Dyer Conjecture: Elliptic curves

The question is... Sometimes straightforward-looking equations – with cubes and squares and suchlike – have nice solutions, in whole numbers or exact fractions, but sometimes they have messy solutions, with infinitely long decimals. Can we find the pattern?

Formal statement 'If an elliptic curve has an infinite number of solutions, its L-series should equal zero at certain points.'

Ludicrously simplistic description 'Brian Birch and Peter Swinnerton-Dyer spotted some intriguing hidden patterns after lots of some tricksy computer number-crunching, and made remarkably intuitive speculations across separate areas of maths to come up with this. It's not a million miles from the Fermat Theorem. A finite whole number of miles, anyway. Or at least, a complicated fractional number of miles. Or at least... '

Consequences if proved '*Deep ramifications for the whole of mathematics... great significance for work on modular forms... making the unsolvable solvable...*'

7. Hodge Conjecture: Algebraic Geometry

The question is... Does there exist, in three dimensions, a number of people greater than zero, who even understand the question?

Formal statement 'Let X be a non-singular complex projective manifold. Then every Hodge class on X is a linear combination with rational coefficients of the cohomology classes of complex subvarieties of X.'

Ludicrously simplistic description 'It's a high-level statement about new kinds of supercharged tools that can link up algebra, geometry, topology and so on. It's not simply a "mathematical Mt Everest", it's like finding *every* Mt Everest that could ever exist. I'd recommend you read Ian Stewart's book on the great maths problems, which has an excellent simplified explanation, readily understood by any reasonably intelligent maths post-doc...'

Consequences if proved '*Important and far-reaching implications... now only scratching the surface... future of mathematics... open up new worlds... one day mathematicians will look back...*'

Other unsolved problems

There are plenty of other major unsolved problems, most with bewildering names. It's easy to make new ones up – simply combine two exotic surnames with plausible jargon. ('*The Tarkovsky-Kurosawa Conjecture on dismorphic chi-cycles*[91]', '*The Bergman–Godard Hypothesis for finite Almodóvar sets*' etc)[92]. Here's a brief list of easy-to-remember real ones.

Goldbach Conjecture Is every even number the sum of two primes – 12=7+5, for instance? We know it's true up to 4,000,000,000,000,000,000... But mathematicians are hard people to convince: just because something has happened four quintillion times doesn't mean it always will. If your partner's a mathematician you'll have a hard time convincing them that they 'always' do something annoying.

Twin primes conjecture Is there an infinite number of primes that are two apart, such as 5 and 7, 101 and 103, etc? The biggest pair we know each have 388,342 digits, but beyond that is a mystery. A similar area of research examines primes that differ by 6 – so-called 'sexy primes', which enables you to move away from number theory and on to discussions about who you fancy most in your Zoom lectures on orthogonal expansions in Hilbert Space.

Three-body problem Thanks to Isaac Newton, we have equations that perfectly predict the way two things move under gravity – the earth round the sun, an apple falling towards earth. But when three bodies are involved things become chaotic and unpredictable, as many love cheats know to their cost, particularly certain lying algebraic topologists we could mention[93]. Cite the gag that the more advanced physics theories, the fewer the number of bodies it can handle. Newton's Laws give up at three. General relativity blows up at two. Quantum theory struggles with one. Quantum field theory can't even cope with a vacuum, in other words, not even *no* bodies.

Langton's Ant Complexity theory is a big new area of maths very useful for modelling complex processes, such as how crowds move around cityscapes. 'Langton's Ant' kind-of models how an ant might move around, following rules that determine its next step according to what it finds on the ground. Some rules seem to mean that, whatever's on the ground, the ant always ends up eventually repeating the same cycle of movements for all eternity – that happens for every pattern we've started it off with, but we don't know if it's true for all possible starting patterns. Which might explain why that wasp buzzing against your half-open bathroom window still hasn't found its way out.

Ulam's Packing Conjecture If you're packing fruit, are oranges a better shape for filling the box than say pears or bananas? A good question that can lead you on to other questions of efficiency, such as why supermarkets throw out so much best-before fruit and veg, and everyone can feel self-righteous.

The perfect Euler brick The 3-4-5 triangle is well-known, but is there a three-dimensional equivalent? A cuboid whose three sides are whole numbers of units, but all the diagonals too, a 'perfect Euler brick'? Nobody knows. This is what a mathematician would mean by a 'perfect house': not a country cottage or stylish urban eco-lodge, but a building with integer-length diagonals. That really would be desirable.

$\aleph_1$: NOTES

1 In the American-speaking world the subject is called 'math'. English speakers can ridicule this. ('*A math? That'th a Roman Catholic thervithe*')

2 Exploit the perceived link between maths and music. Say grand things like '*Music is not frozen architecture, as the cliché claims. It is aural mathematics. Bach is just group theory. John Coltrane is algebraic topology. Javanese gamelan is the Birch/ Swinnerton-Dyer conjecture on rational solutions to elliptic-curve equations*.' All rubbish, of course, but it'll fool non-mathematicians who'll think you're serious, and amuse mathematicians, who'll think you're joking.

3 UK edition. The US equivalent is Illinois Avenue. Trafalgar Square is indeed square, and with sides of 110m, is pretty close to a hectare (100m x 100m) in case you need to visualise how big one is.

4 The best property to buy depends on how you measure it. Likely return on investment, but over how long? With how many houses or hotels? Landing probability and property building varies through the game. Hannah Fry and Matt Parker's YouTube video discusses things in more detail, but for the purposes of impressing someone, you can claim any old property is '*mathematically the most productive one to buy*'. ('*Whitechapel Rd/ Vine St/ Trafalgar Square/ Mayfair is actually the most lucrative because it's the highest return ratio/ most-landed on/ easiest to get a high-rent hotel on/ most likely to actually win you the game*' etc.)

5 'Hommy-o-morfick': the same essential thing, in the sense of being the same essential shape, in the sense that if the two shapes were made of clay, you could squidge one to look like the other without having to close up holes or create any new ones, or cheat by joining or cutting anything. So, a coffee cup and a doughnut are homeomorphic because they have one hole each (the cup's handle being the 'hole'). But a coffee cup and a wine glass aren't, nor are a doughnut and a banana. You'll be getting some ideas of your own now – mainly, about the typical mathematician's diet.

6 The 'Moving Sofa' problem. The sofa's footprint can be any shape, not necessarily rectangular. If it's like an old telephone handset you can make it remarkably large compared to the narrow corridor whose 90-degree bend you're manoeuvring round, but larger solutions are known to be possible. If you hear mathematicians talking about 'Gerver's Sofa' or 'Hammersley's Sofa', you'll know that they're not talking real furniture, but imaginary, idealised shapes. Another sofa-based enigma is why the one that seemed so comfy in the shop is unsittable without backache when you finally get it home. We don't have an answer to this. Maybe it got deformed while pushing it round a bend.

7 We know that a group of 49 people certainly will contain five friends and five strangers, and that a group of 42 cannot, but don't know what holds for the numbers in between. The 'party problem' is a special case of Ramsey Theory, after the 20th-century mathematician Frank Ramsey, which looks at possible sub-combinations of complicated collections of things. It often involves working with numbers of stupendous size, of which Graham's Number, g_{64}, is an example. If anyone refers to numbers as 'astronomical', you can snort and smile and say, '*Hah! Ramsey Theory! Makes astronomy look like finger-counting!*'

8 The 'P vs NP' problem – see Unsolved Problems chapter. Doesn't seem likely, given the number of times satnavs maroon drivers in rivers, on railway lines etc.

9 The 'Riemann Hypothesis' – another one in the Unsolved Problems chapter. We don't know if it's true or false. Maybe it's true, but unprovable. Perhaps the most celebrated unknown in maths, along with the unresolved question of whether there's space in the market for another popular-maths book on the subject when Marcus du Sautoy's was so good.

10 The author Douglas Adams coined the word 'bistromathics' in his *Hitchhiker's Guide* series to describe the cumbersome computational methods needed to divide a restaurant bill between participants.

11 Quadratic equations come in handy if you want to calculate the speed of a vehicle, relative to you, as it nearly knocks you off your bike while overtaking you. You need to recognise the musical interval described by the drop in pitch of the engine, or perhaps horn, as it passes (the 'Doppler Effect'). You also need to know the ratio of frequencies this interval represents. That ratio will be equal to (V-v)/(V+v), where V is the speed of sound and v the speed of the vehicle relative to you (strictly 'velocity'). That gives you a quadratic equation, which you can solve to give you v, if you know V. So, if you know your own speed ('velocity...') you can work out what actual speed the vehicle was doing. Unfortunately, if you try doing that in your head while cycling on from your near-miss, you may get distracted and be unable to avoid the next close- passer.

12 *Countdown* is a British TV programme that involves adding up and multiplying.

13 This apparent paradox almost brought maths down. Bertrand Russell noticed it in 1901 as he was trying to lay down a foolproof foundation for the rules of maths using set theory – a foundation that seemed undermined by this potential banana-skin that could have blown up in his face, and left egg on it. Luckily mathematicians worked

together, and came up with a new rule that said this wasn't a paradox, so that was OK, and maths is true after all.

14 Group theory looks at recurring patterns in when collections of objects undergo systematic change – how Rubik Cubes behave when you twizzle them, for example, or how many iterations of a prog-rock band line-up it takes before they reform with the original members.

15 Graph Theory, confusingly, is nothing to do with the graphs you draw in school maths that look like hilly skylines. It sort-of examines the way points can be joined up, such as the classic (and first graph-theory) question about bridges to the 18th-century central island in Königsberg. Was it possible to go for a walk using all seven bridges to the island without having to use one twice? No it wasn't, as Euler proved, but now it is because they've built more bridges, not that most of us can visit easily to find out because it's now Kaliningrad, an exclave of Russia.

16 Topology is the study of shapes. Imagine a world in which everything is made of rubber or plasticine, but you're not allowed to cut, glue or pierce anything. It turns out lots of things can be literally pressed into service as something else (such as making a dish from a bowl) while others can't (such as making a coffee mug from a dish, because one has a hole – the handle – and the other doesn't). Actually trying to make coffee mugs, bowls or plates from these materials is not recommended.

17 These people are actually prominent footballers, which works in two ways. First, it's easy to find a list of plausibly exotic names, which are refreshed regularly, simply by browsing the squads of any English Premier League side. Second, if anyone gets suspicious and tries a Google search on your purported theorem, their search results will be clogged with references to the footballers, and they can't be sure that there really is a theorem of that name but it's just way down the list of results.

18 The Four Colour Map Theorem was the first theorem to be proved with the help of a computer, in 1976 by Kenneth Appel and Wolfgang Haken. They showed that all possible sensible maps had to be one of 1,843 basic configurations. However, they didn't have enough crayons or time to check each of those individually, so got a computer to do it.

19 'E to the eye pie plus one equals zero'. Often cited as the most beautifully succinct equation in maths, a sort of equivalent to $E=mc^2$ in physics, or 'survival of the fittest' in biology, or 'try turning it off and then on again' in computing.

20 The 'Julia Set' '*pops up in complex dynamics*' and is '*all to do with fractals*' and '*chaotic behaviour*' (you say). Essentially, if you try to do something with a member of the Julia Set of a function, you get a chaotic result – so if the function is 'appoint them as manager', then the Julia Set consists of the managers who, if appointed, will result in chaos. The Fatou Set is the opposite – managers who result in stability. If you plot the Julia Set you get amazingly intricate, never ending patterns-within-patterns. It was named after a mathematician called Julia, but it was a man – Gaston Julia.

21 There appears to be some debate about what kind of torus the human body is equivalent to – that is, how many holes it has. The digestive tract makes it clearly not homeomorphic to a sphere, but the topological nature of the ears, nostrils etc complicates things.

22 Accounts of alien abductions reveal their implausibility here. After the medical examination but before the tour of the ship, the abductees would surely be grilled by the extra-terrestrials on the stuff that really matters. *Have you earthlings solved the G'l'hrg–Z'zxvnt Conjecture – or as it is apparently known by your more intelligent life-forms, the Riemann Hypothesis?* Such things are always missing, putting the truth of the stories in serious doubt.

23 He devised a way of encoding every possible mathematical statement as a single number. It's a bit like the joke of the comedian who goes on stage and says '34!', and everyone titters. Then he says '92!', and everyone chuckles. One puzzled audience member asks their neighbour what's going on. Ah, the neighbour says, we've seen him so many times we know all his jokes, so to save time he's numbered them.' The comedian says '263!' and everyone laughs uncontrollably. 'Ha ha! Brilliant!' says the neighbour. 'We haven't heard that one before'. Well, sort of.

24 If you do set your story in the Yorkshire Dales then it's probably a Wensleydale, which occasionally produce black lambs. Black Wensleydales, in the language of sheep farming, are called 'Black Wensleydales'.

25 The explorer had been trained in maths, but his calculations were terrible, and he fiddled his figures to get the results he wanted. He thought the West Indies were the East Indies, and he got Japan's position wrong by 8,000 miles.

26 Top and bottom divide by 7, so it should be reduced to 4/13. This is the sort of thing that makes us smile.

27 The Secretary Problem goes like this. A company is interviewing a series of hopefuls to fill a position. You know how many interviewees will be coming. The catch is that after each interview, they either have to reject them forever, or offer them the post. With the optimal stopping rule, you have a 1/*e* (so, about 0.36) chance of getting the best candidate. The rule is this: if there are *n* applicants, then interview the first *n*/*e* of them, but reject them all. (So if there are 27 candidates, you interview the first 27/*e*, in other words, nine, to the nearest whole person.)

Continue interviewing, and offer the job to the first remaining candidate who is better than the best so far. (Or the last candidate, if necessary.) Obviously things are not as complex in real life. In real life the company has already decided to give the job to their CEO's friend's daughter.

28 You can divide 7 into it, so it's not prime. Ditto.

29 A 'perfect' number is equal to the sum of anything that divides into it, like 6 (=1+2+3). Nothing divides into a prime number except 1, so the only perfect prime number is 1, which is not a particularly helpful fact. This is the sort of thing that makes us laugh.

30 Pi (and hence half a pi) is irrational, implying that it's irrational to hope for a perfect family. This is quite a subtle joke, so we'll be too busy working it out to smile or laugh.

31 Paradoxes fall into one of two types: those that are genuinely a paradox, and those that, paradoxically, aren't.

32 Comedic timing is ruined by, for example, having to look up an endnote.

33 Bertrand Russell's 1919 book *Introduction to Mathematical Philosophy* has quite a bit about socks – specifically, the problems in choosing from pairs of socks (which have no left-right difference) as opposed to shoes (which do). From an infinitely large drawer, of course. You can drop this into a conversation, noting that this is a way in to thinking about the 'axiom of choice', and it might distract people from noticing that *your* socks don't match.

34 An extreme example is Pirahã, an Amazonian language with about 400 native speakers, which doesn't appear to have any specific number-words at all. They only have terms meaning roughly 'not that many' and 'quite a lot', but they're both *hoi*. Rendering the song 'Ten green bottles' into Pirahã clearly presents challenges for the translator.

35 In set theory, curly brackets are used to list the members of a set. For instance, {curly, angle, round, box} is the set of brackets on the standard computer keyboard.

36 '*Division by zero is not defined*', you should say airily, '*so it's meaningless, like asking what's a lemon divided by an umbrella*', or similar.

37 An interesting one, this, because it can be 0 or 1, depending on which direction you start from. If someone has an opinion on which is the better answer, maintain it's the other one, with a dismissal such as '*you've been talking to too many set theorists/ algebraic topologists/ games teachers forced into teaching first-year maths*' etc.

38 So 1=0.999..., and this visual anomaly upsets quite a few people, who refuse to accept that the two numbers are *exactly* the same thing. A good thing to remember if you want to upset someone, then.

39 Amicable numbers add up to each other's divisors – for example, 220 and 284 are amicable because 284=1+2+ 4+ 5+10+11+20+22+44+55+110 (the numbers which divide into 220) and 220=1+2+4+71+142 (the numbers which divide into 284). The other definitions are more complicated so it's unlikely anyone would challenge you to explain them. All you need remember is that being amicable doesn't mean being friendly, or sociable, or happy.

40 An approximation is 22/7; a closer approximation is 355/113, which impressively comes within 0.000009% of the actual value ('equivalent to half an inch in a hundred miles', you can say in awe). There are plenty more, such as 103993/33102, but they tend not to crop up as convenient hooks in exam questions – though of course any real mathematician, especially one who sets exam questions, will take that statement as a challenge.

41 The longest calculated version of pi so far (as of 2021) is to 50 trillion digits. It would take 300,000 years to read it out. However, there are ways of working out specific digits further along the expansion. We know for example that the quadrillionth digit of pi is 0, but don't know what's on either side.

42 The easiest way to characterise e might be that if you plot the graph of $y=e^x$, the gradient of the curve is the same as the value at that point – the equivalent of an uphill whose steepness is always the same as the altitude. Which means it gets steep very quickly. And the rate at which the gradient increases is also the same as the value at that point. In fact, the rate at which the rate at which it gets steeper is also the same, as is the rate at which the rate at which the rate which... you get the drift, which is exactly what people tend to do when you tell them this: off, to sleep.

43 '*Obviously, it's equal to the square root of five plus one, all divided by two*', you can say. If challenged, start folding up sheets of A4 paper in half and half again, as if this proves it.

44 The books have 40 lines on each page and 80 characters on each line, hence 410 x 40 x 80 = 1,312,000 characters each. They're in Spanish, so Borges reckoned on 25 possible characters (a 22-letter alphabet and three punctuation characters). The total number of books is therefore 25^1,312,000, a number about two million digits long.

45 A former favourite situation of maths problem setters was a drunk taking a random walk, for which you had to calculate various probabilities such as him (it was always a him) getting back to where he started, or falling off a cliff edge, and so on. This scenario seems less acceptable now, as sensibilities have changed. ('*Quite right too, we shouldn't be glamourising the consumption of alcohol. I drink far less than I used to. Goodness knows how I finished my algebraic topology PhD. More wine, anyone...?*')

46 Tens of thousands of creditors were left out of pocket and the total money that disappeared was over $1bn. There were nearly two dozen 'Ponzi schemes' operating, a type of pyramid scheme where new investors' money is used enitrely to pay interest to the first few, lucky, investors. Obviously you or I would never have fallen for such a thing. Did you also know that the Thai national anthem was originally 'O, wattana, Siam!', and sung to the tune of the British national anthem? Try it.

47 Nobody seems to know the origin of this phrase, so you can claim it 'goes back to Shakespeare/ the Bible/ ancient Egypt' or whatever. Some keen maths bloggers who have analysed the stats suggest that, if you wait for an actual London bus, then about 1 in 10 times two will come along at once, so there seems a fair amount of truth in the saying.

48 Wensleydale, perhaps. Unusually, that dale is named after a now-obscure village, Wensley, not its river, which is the Ure. The Ure is the river that continues on through York, except by that time it's called the Ouse, and nobody knows why. Which is nothing to do with maths, but useful if you want to change the subject.

49 "Is not the whole land before thee? Separate thyself, I pray thee, from me: if thou wilt take the left hand, then I will go to the right; or if thou depart to the right hand, then I will go to the left. Understandeth thee not? Look, Lot, it be obvious, for crying out loud. You divide, I choose. Then it's fair. Get it? Excellent." (Genesis Chapter 13 Verse 9, King James Version first draft.)

50 The genuine mathematician will take this as a challenge. 'But is this true? If a cake of diameter d and height h takes times *f*(*d,h*) to bake and *c*(*d.h,n*) to cut for n people, then...'

51 Benford's Law doesn't really work for election results because the range of numbers involved is usually not wide enough – electoral districts tend to be similar sizes, and not vary over several orders of magnitude (like from one to a million) for instance. However, this didn't stop some people from inaccurately invoking Benford to challenge Joe Biden's 2020 US election victory. But then, not many things did.

52 In case any banks write in to complain that we're exaggerating, it's not quite as bad as that. The actual figure is only 6,932 years.

53 In English the most common words roughly obey the law: 'the' is tops at 7% of all words, 'of' is next as 3.5%, and 'and' third at 2.7%. You can have a dig at your least favourite writers here by alleging they overuse certain words in a Zipf-defying way. The book *Nabokov's Favorite Word is Mauve* examines authorial quirks like this.

54 He also dealt with the theory of issues such as a candidate dying mid-election, and how this could be most fairly dealt with. However, Arrow never considered referendums being swayed by figures on the side of a bus.

55 As opposed to those people who think they're an expert on urban planning just because they don't have a car.

56 The coin's early incarnations featured a thrift plant. Oh, very clever.

57 No.

58 In the UK, many speed cameras work by calculating your average: they know the time you passed camera A and camera B, and the distance between them, and divide one by the other to give your average (ie, 'arithmetic mean') speed between them. Which means, you can confidently say, it's perfectly possible to break the speed limit between them – to overtake, for example – so long as you slow down enough so as to pass the second camera at a time that brings your average down. We don't recommend this, partly because you might get your mental arithmetic wrong, but mainly because it might be the author that you're passing at speed, on his bike.

59 The series of numbers, first noted by Leonardo of Pisa in the late 1100s, where each is the sum of the two previous (it starts with two 1s). That is, 1, 1, 2 (=1+1), 3 (=2+1), 5 (=3+2), 8 (=5+3), 13 (=8+5) and so on. One of its properties is that it gives the population of immortal rabbits each generation, if every breeding pair produces one baby. A property of the series is that, as you go further down it, dividing a term by the one before gets closer and closer to the Golden Ratio, ϕ. This is the sort of thing you can say is 'obvious why, once you see it' It's not really, of course..

60 Named indeed after Plato, the Greek philosopher who worked around 400BC. He speculated that the four elements of earth, air and fire and water were made of the cube, octahedron, tetrahedron and icosahedron respectively, and reckoned the dodecahedron was reserved for divine use. He also wrote extensively about the nature of love, infatuation and sex, hence 'Platonic relationship', which people will be much more interested in talking about.

61 You could drop into the conversation here 'Penrose tiling', after the British mathematician Roger Penrose: an ingenious way of tiling a wall or floor that looks as though patterns repeat but, in fact, they never do. Name-drop a few places that are decorated with the pattern as if to suggest you've been to them ('*San Francisco's Transit Center... University of Western Australia... Helsinki... Andrew Wiles Building in Oxford, obviously...* ')

62 In practice, not quite 50–50: there is a small chance the coin will come to rest on its edge, and anyway slight inconsistencies in shape mean one side will be slightly favoured. When mathematician John Kerrich was a prisoner

of war in Denmark in 1940, he did exactly what you'd expect of a our kind – not try to escape, but use his time to test Bernouilli's Law of Large Numbers by tossing a coin 10,000 times. Both Kerrich and the Law came out unscathed.

63 Morissette, A. [1995] Jagged Little Pill. New York: Maverick Records

64 With large rollovers, you can expect to make a profit if you can buy tickets for every possible number. This happened a few times in various lotteries, but lotteries now make sure it's not possible, or not practical, to block-buy tickets in the quantities required. The best way to look at lottery tickets is not the possibility of winning, which is obviously of negligible probability, but as buying the chance to dream of doing so. Therefore, the best strategy is only to spend the minimum amount, and buy the ticket as early as possible, thus maximising your fantasy time. You also have the maximum chance to tell everyone how generous you would be to your friends if you won, then sit back and wait for them to buy you a drink in gratitude.

65 In the standard version of the puzzle, a car is the prize, and the non-prize doors open to reveal goats. This sounds unlikely, because you'd just choose the widest door.

66 At least you know it's a safe bet that Oxford and Cambridge make it through to the final.

67 A busy UK chain of snack-food bakers. Popular with mathematicians, partly because of the breakfast deal with bacon roll and coffee, but also because they provide plenty of data for queueing theorists.

68 The geometric mean is obtained by multiplying together all n terms and then taking the nth root. So the arithmetic mean of 3 and 12 is $^{15}/_{2}$= $^{71}/_{2}$, but the geometric mean is the square root of 36 (3x12) which is 6. The geometric mean is always less than the arithmetic mean (unless every item in the list is the same, in which case every average is the same). You can make the same accusation of political bias by newspapers when they mention house prices over which one they selected to reinforce their message of more/less affordable.

69 Furthermore, as the Arctic hunter's saying supposedly goes, it's not the polar bear you see that gets you.

70 Maybe even the same one you were using to discuss the Golden Ratio previously.

71 Surprisingly at first glance, zero factorial equals one. You might be able to use this to your advantage, for example '*The number of times I've been sacked from a job for gross misconduct is 0!*'

72 You can actually represent all the combinations of any number of sets using Venn diagrams, but they get very complex, with long extra snake-shapes threading in and out of the three circles of the familiar three-set diagram. The three-set, three-circle Venn diagram is enough for all genuinely useful purposes – social media memes, in other words.

73 Q: What do you get if you rearrange the letters of 'Banach-Tarski Paradox'? A: 'Toxic Arab Panda Shark'. Q: What do you get if you rearrange the letters of 'Toxic Arab Panda Shark'? A: ' Banach-Tarski Banach-Tarski Paradox Paradox'.

74 One study suggested that 93% of US drivers rated themselves as 'above average', but only 69% of Swedes did. If average refers to 'median', then of course it's impossible for all the drivers to be right – by definition, the proportion above the middle driver must be less than 50%. If it refers to 'mean', however, it's possible. For instance, a set of seven zeroes and 93 ones has an average of 0.93, so all 93 of the ones are above average. Unlikely though.

75 'The process is very efficient, very smart, but hard to explain succinctly' (you can say) 'but it's of vital importance in cryptography and internet security – deriving key-pairs in the RSA public-key encryption method for example, *everyone* knows that'

76 For most people, good old common sense ended with the advent of smartphone-based social media, or about the time they turned 50, whichever is earlier.

77 An example of a 'Winograd Schema' – a sentence which a computer would find impossible to understand without interlligent use of background knowledge. Such schemas are important in AI-spotting challenges where you have to decide if you're having a keyboard conversation with a human being or a well-drilled robot – a situation familiar to anyone who's tried to contact a train company via their Twitter feed.

78 Either in the 'pampered millennial' sense, or the meteorological sense.

79 BBC weather forecaster who either did or didn't fail to forecast the Great Gale of 1987, depending on who you believe.

80 There are no endnotes for this chapter.

81 The Blancmange Function looks nothing like Teiji Takagi, who described it 1901, but a bit like a blancmange. Its infinitely crinkly contour gives it the strange property of being continuous (so you can skate along it, even if you have very small skates) but not being differentiable (so you can't tell what the gradient is at any point – you have no idea whether you're going uphill or downhill).

82 The genuine subject of my friend Ron's PhD. He has little idea what it was all about now, either.

83 If you want to diss a media mathematician, you can paraphrase the things snooty music critics say about artists they can't bear but who enjoy media success – things which can be taken either way: '*Yes, she's become very popular, hasn't she? Especially with people who know nothing about the subject. You've got to admire her self-belief. Does what she does very well. There's no doubting the extent of her talent. Genius doesn't begin to describe her...*' etc. The author is not referring to anyone mentioned in this book. Obviously.

84 Such as the episode where Homer apparently finds a counterexample to Fermat's Last Theorem, chalking up on a blackboard that $3987^{12}+4365^{12}=4472^{12}$. The joke is that calculators can't handle such big numbers and may show the sum as 'correct' to their limit of accuracy. It isn't, of course: just a near-miss.

85 The next best-known Pythagorean triples are 5–12–13, 8–15–17, and 7–24–25. There's an infinite number of them, even discounting multiples (eg 6–8–10, which is twice the 3–4–5). However, that's not the most interesting thing about right-angled triangles. It's the fact that many people find the word 'hypotenuse' intrinsically funny.

86 Mathematicians routinely talk about 'Euclidean geometry' to mean the sort you learn at school, and that engineers use to design planes, buildings, missiles and so on. However, there are alternative ways to set up the rules, like drawing your shapes on a sphere instead of a flat sheet of paper. Some of these non-Euclidean systems actually prove to be more accurate for some purposes, such as astronomy: Einstein's work on relativity showed that the universe is warped. Depending on whether it's bent inwards or outwards, that means that strictly, the angles of a triangle always add up to less than, or more than, 180 degrees, respectively.

87 '*Of course*', you can say, '*Wiles got there by proving the Taniyama-Shimura conjecture, relating elliptic curves on the field of rationals to modular forms. So it's now called the Modularity Theorem, as it's no longer a conjecture*.' If anyone demands you explain any of this. shrug, smile, and direct them to Simon Singh's very good book on the whole Fermat/Wiles thing, stressing that it's 'very readable', with the implied subtext of '...even if you don't understand modular forms, which Simon and I do'.

88 The tree (or at least a tree) is still there (well, the original blew down in a storm in 1820 and what you see now grew from the remains). It's in Woolsthorpe Manor, near Grantham in Lincolnshire, and is open to the public. Some apple pips from the tree went into space in 2015 with British astronaut Tim Peake to see if they could germinate up there. They did eventually come back down to earth, confirming Newton's thought experiment.

89 That's a joke, like almost everything else in this book. Galois Theory in fact links field theory and group theory: areas of maths looking at how patterns change or are preserved when you alter them (such as on a Rubik's cube) and the consequences for finding useful solutions of equations, which sometimes follow group-like behaviour. As usual, if someone demands more explanation, talk casually about '*Ian Stewart's book... some excellent YouTube videos... Wikipedia page... Wolfram Math World website... some episode of* The Simpsons...' as if you're familiar with them all.

90 Rishi Sunak, who took over as UK Chancellor of the Exchequer in 2020 shortly before the pandemic, did 'teach' a primary school class in Watford for a few minutes by Zoom as part of a 2021 PR stunt. He talked about basic multiplication and geometry. Sunak's academic background was the politician's standard-pattern Politics, Philosophy and Economics at Oxford, so he would have studied maths for economists. If relating this fact, leave a small pause after 'maths' and add '...*for economists...*' with a knowing smile, raised eyebrow, and the tone of voice you'd use in faint-praise damning such as 'that's really very, very good indeed... *for a beginner...*'.

91 There's no such thing as a 'dismorphic chi-cycle', but by the time you read this, some mathematician will probably have investigated them. If not that, then some dodgy engineering startup will have invented one on Kickstarter, promising in their video to 'reinvent the bike', only to disappear without trace but with the money.

92 These are names of film directors, of course, not mathematicians, but a good area to mine if you need a clever-sounding exotic name on demand. If you're a mathematician who for some reason needs to curry favour with arthouse film buffs, you could try it the other way round: '*Have you seen Terence Tao's latest? Remarkable individual vision, I thought. Challenging imagery though. I'm quite a fan of Maryam Mirzakhani's work, and all that wonderful Iranian tradition...*'

93 A joke, created for the book. I have no problem with algebraic topologists. Not in in any way, shape or form.

About the author

Rob Ainsley was once the top mathematician of his age group. That was only when he was 17 years 167 days old, though. And that was mostly for alphabetical reasons. But he did maths at Oxford, occasionally, and then trained to be a maths teacher, making a positive and lasting contribution to British mathematics education by not becoming one. After working as a computer and classical music journalist for three years – out of the total of fifteen he was employed to do so – he did a music degree. This demonstrates the link between ability in maths and music: he didn't know what he was doing half the time in his maths degree, either. Rob is now a cycling writer and lives in York, but does more cycling than writing. Except in lockdowns, when it's the other way round.